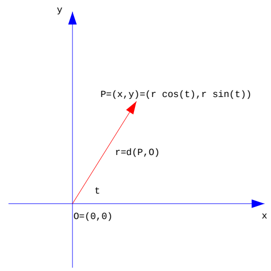
CYLINDRICAL AND SPHERICAL COORDINATES — **Maths21a, O. Knill**

POLAR COORDINATES. A point (x, y) in the plane has the **polar coordinates** $r = \sqrt{x^2 + y^2}, \theta = \text{arctg}(y/x)$. We have

$$(x, y)(r\cos(\theta), r\sin(\theta))$$

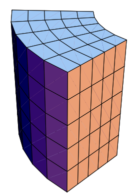


Footnote: Note that $\theta = \text{arctg}(y/x)$ defines the angle θ only up to an addition of π. The points (x, y) and $(-x, -y)$ would have the same θ. In order to get the correct θ, one could take $\arctan(y/x)$ in $(-\pi/2, \pi/2]$ as Mathematica does, where $\pi/2$ is the value when $y/x = \infty$, and add π if $x < 0$ or $x = 0, y < 0$. In Mathematica, you can get the polar coordinates with
$(r, \theta) = (\text{Abs}[x + Iy], \text{Arg}[x + Iy])$.

EXAMPLES OF CURVES IN POLAR COORDINATES.
EXAMPLE 1: $r = 1$ circle
EXAMPLE 2: $r = |\cos(3\theta)|$ rose
EXAMPLE 3: $\theta = \pi/2$ positive y axes.
EXAMPLE 4: $r = 1/\cos(\theta)$, $\theta \in [-\pi/2, \pi/2]$ line $x = 1$.

CYLINDRICAL COORDINATES. Use polar coordinates in the x-y plane and leave the z coordinate.

$$(x, y, z) = (r, \theta, z) = (r\cos(\theta), r\sin(\theta), z)$$

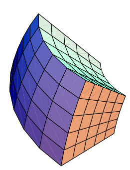


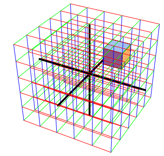
EXAMPLES OF SURFACES IN CYLINDRICAL COORDINATES.

EXAMPLE 1: $r = 1$. cylinder
EXAMPLE 2: $r = z$. double cone
EXAMPLE 3: $\theta = 0$ half plane.
EXAMPLE 4: $r = \theta$ rolled sheet of paper
EXAMPLE 5: $r = 2 + \sin(z)$ surface of revolution

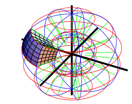
SPHERICAL COORDINATES. Spherical coordinates use ρ, the distance to the origin as well as two angles: θ the polar angle and ϕ, the angle between the vector and the z axis. We can describe a point as

$$(x, y, z) = (\rho\cos(\theta)\sin(\phi), \rho\sin(\theta)\sin(\phi), \rho\cos(\phi))$$





$$(x, y, z) = (\rho\cos(\theta)\sin(\phi), \rho\sin(\theta)\sin(\phi), \rho\cos(\phi))$$



EXAMPLES OF SURFACES IN SPHERICAL COORDINATES.

EXAMPLE 1: $\rho = 1$. sphere.
EXAMPLE 2: $\phi = \pi/2$ single cone
EXAMPLE 3: $\rho = \phi$ apple
EXAMPLE 4: $\rho = 2 + \cos(3\theta)\sin(\phi)$ bumpy sphere

LATITUDE AND LONGITUDE. If we fix ρ, then two given angles $\theta \in [0, 2\pi)$ and $\phi \in [0, \pi]$ describe a point on the sphere. θ is the **longitude** und ϕ is related to the latitude, but the latitude has the zero at $\phi = \pi/2$. For example, the latitude of Boston is comparable with the latitude of Rome in Italy:

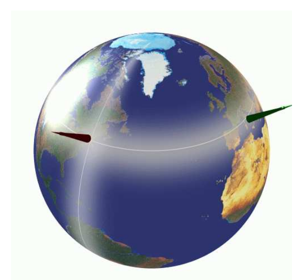
Rome, Italy;	41:53:00N	12:30:00E
Boston, USA;	42:21:24N	71:03:25W

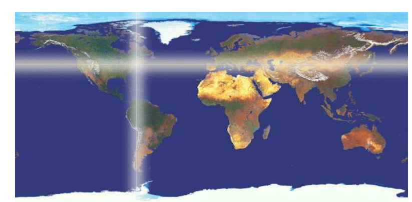
PROBLEM Estimate the distance between the points B (Boston) and R (Rome) in Euclidean space. By nature, the coordinates of these points are given in spherical coordinates, so that we have to convert them into rectangular coordinates. The radius is $r = 6582$. We assume the center of the earth is $(0, 0, 0)$. Assume that the x axis passes through Boston. To simplify the computations, we assume that both Rome and Boston have latitude 45 and that their longitude differs by 90 degrees. We determine first the Euclidean coordinates of B and R:

$B = (\rho, \theta, \phi) = (6582, 0, \pi/4)$ spherical B=(x,y,z) =

$R = (\rho, \theta, \phi) = (6582, \pi/2, \pi/4)$ spherical R=(x,y,z) =

What is the distance between these points? With distance we mean the distance between the two points and not the distance along the earth surface.





VISUALIZING SURFACES IN OTHER COORDINATES.
- Computer algebra system like Mathematica (Site licence for Harvard)
- Grapher (OSX Tiger Utility) Class demo
- Graphics Calculator (OS X and Windows) Class demo
- (www.pacifict.com)
- Online applets
- Graphics Calculators