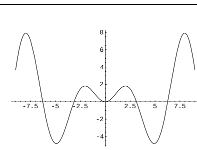
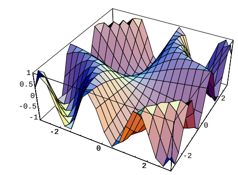
FUNCTIONS OF THREE VARIABLES **Maths21a, O. Knill**

FUNCTIONS OF ONE VARIABLES. A function of one variable $f(x)$ assigns to a variable x a real number $f(x)$. Example: $f(x) = x\sin(x)$. We can visualize a function by its **graph** $\{(x, y),\ y = f(x)\}$ which is a curve in space.



FUNCTIONS OF TWO VARIABLES. A function of two variables $f(x, y)$ assigns to two variables x, y a real number $f(x, y)$. Example: $f(x, y) = \sin(xy)$. It could describe for example the temperature distribution on a plate.
In the same way as functions of one variable were visualized by drawing the graph $y = f(x)$ in space, we can visualize a function of three variables as a graph $\{(x, y, z) \mid z = f(x, y)\ \}$ in space.

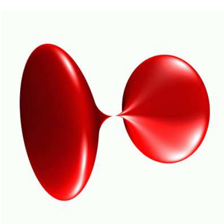


FUNCTIONS OF THREE VARIABLES. A function of three variables $g(x, y, z)$ assigns to three variables x, y, z a real number $g(x, y, z)$. Example: $g(x, y, z) = \sin(xyz)$, temperature distribution in space. We can no more draw a graph of g. But we can visualize it differently by drawing surfaces $g(x, y, z) = c$, where c is constant.

SURFACES. Many surfaces can be described as a functions $g(x, y, z)$: Examples.

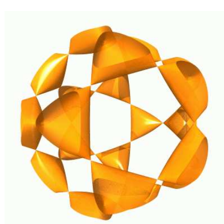
Graphs: $g(x, y, z) = z - f(x, y)$. If $g(x, y, z) = 0$, then $z = f(x, y)$ and the surface is a graph of a function of two variables.
Planes: $ax + by + cz = d$ is a plane orthogonal to the vector $\vec{n} = (a, b, c)$. The equation says $\vec{n} \cdot \vec{x} = d$. If a point $\vec{x}_0$ is on the plane, then $\vec{n} \cdot \vec{x}_0 = d$. The equation can also be written as $\vec{n} \cdot (\vec{x} - \vec{x}_0) = 0$ which means that every vector $\vec{x} - \vec{x}_0$ is orthogonal to $\vec{n}$.
Quadrics: If $f(x, y, z) = ax^2 + by^2 + cz^2 + dxy + exz + fyz + gx + hy + kz + m$ the a surface $f(x, y, z) = 0$ is called a **quadric**. Below are some examples.

QUARTIC



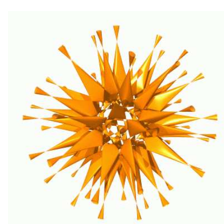
$p(x, y, z)$ degree 4 polynomial in x,y,z

SEXTIC



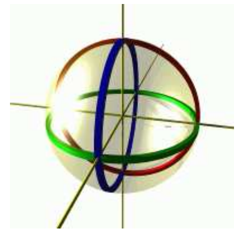
$p(x, y, z)$ degree 6 polynomial in x,y,z

DECIC



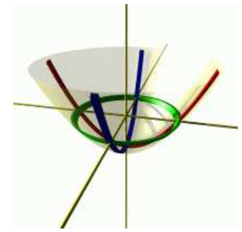
$p(x, y, z)$ degree 10 polynomial in x,y,z

SPHERE



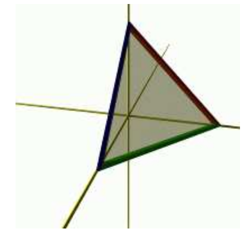
$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$

PARABOLOID



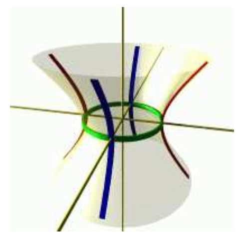
$(x - a)^2 + (y - b)^2 - c = z$

PLANE



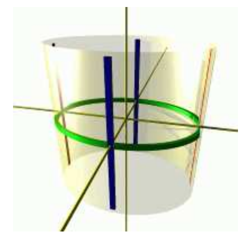
$ax + by + cz = d$

HYPERBOLOID I



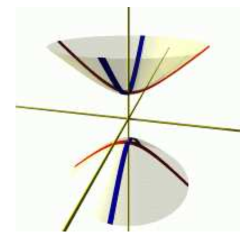
$(x - a)^2 + (y - b)^2 - (z - c)^2 = r^2$

CYLINDER



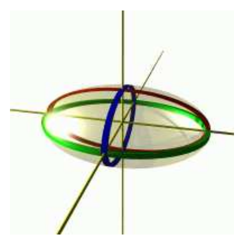
$(x - a)^2 + (y - b)^2 = r^2$

HYPERBOLOID II



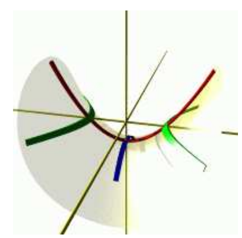
$(x - a)^2 + (y - b)^2 - (z - c)^2 = -r^2$

ELLIPSOID



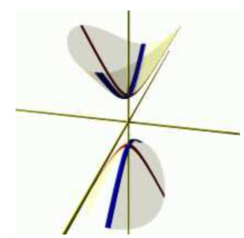
$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$

HYPERBOLIC PARABOLOID



$x^2 - y^2 + z = 1$

DEFORMED HYPERBOLOID



$x^2/a^2 + y^2/b^2 - z^2/c^2 = 1$

TRACES. To draw surfaces, it helps to look at the **traces**, the intersections of the surfaces with the coordinate planes $x = 0, y = 0$ or $z = 0$.

INTERCEPTS. Other points to look at are the **intercepts**, the intersections of the surface with the coordinate axis. The traces are shown in the pictures of the quadrics above.

For example: for the **one-sided hyperboloid** $x^2 + y^2 - z^2 = 1$ (called HYPERBOLOID I above), the z-trace is $x^2 + y^2 = 1$, a circle, the x-traces $y^2 - z^2 = 1$ is a hyperbola as well as the y-trace $x^2 - z^2 = 1$.