**7/22/2008 DOUBLE INTEGRALS** **O.Knill,Maths21a** This is part 1 (of 2) of the weekly homework. It is due July 29 at the beginning of class.

SUMMARY. We sometimes $dA = dxdy$ for the **area element**.

- $\int\int_R f\ dA = \int_a^b \int_c^d f(x,y)\ dydx$ is called a **double integral** over a rectangle $R$.
- $\int\int_R f\ dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\ dydx$ double integral over a **type I region**.
- $\int\int_R f\ dA = \int_a^b \int_{h_1(y)}^{h_2(y)} f(x,y)\ dxdy$ double integral over a **type II region**.
- $A(R) = \int\int_R 1\ dA$ is called the **area** of $R$.
- $\frac{1}{A(R)} \int\int_R f\ dA$ is called the **average value** or the **mean** of $f$ on $R$.
- For $f \geq 0$, the integral $\int_R f\ dA$ is the volume of the solid over R bounded below by the xy-plane and bounded above by the graph of $f$.

## Homework Problems

1) (4 points) Calculate the iterated integral $\int_1^4 \int_0^2 (2x - \sqrt{y})\ dxdy$. Can you interpret it as a volume of a solid? If not, can you express the result in terms of two volumes?

2) (4 points) Find the area of the region

$$R = \{(x,y) \mid 0 \leq x \leq 2\pi, \sin(x) - 1 \leq y \leq \cos(x) + 2\}$$

and use it to compute the average value of $f(x,y) = y$ over that region.

**Remark.** You will use here the integral $\int_0^{2\pi} \sin^2(x)\ dx$ treated in class.

3) (4 points) Find the volume of the solid lying under the paraboloid $z = x^2 + y^2$ and above the rectangle $R = [-2,2] \times [-3,3] = \{(x,y) \mid -2 \leq x \leq 2, -3 \leq y \leq 3\}$.

4) (4 points) Calculate the iterated integral $\int_0^1 \int_x^{2-x} (x^2 - y)\ dydx$. Sketch the corresponding type I region. Write this integral as integral over a type II region and compute the integral again.

5) (4 points) Evaluate the double integral

$$\int_0^2 \int_{x^2}^4 \frac{x}{e^{y^2}}\ dydx\ .$$

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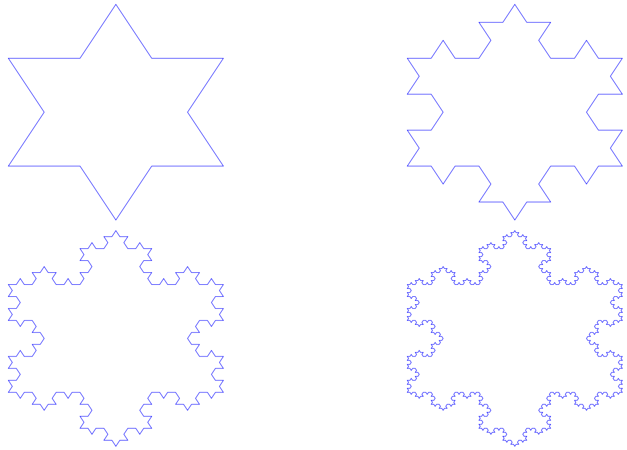
**Remarks**: (You don't need to read these remarks to do the problems.)

Area computations can also be done for regions which are not smooth. Here is an example of a region defined iteratively. In each step we can compute the area. What is the area in the limit



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## Challenge Problems

(Solutions to these problems are **not** turned in with the homework.)

1) Let $M$ be a polygon in the plane where each edge is at a lattice point. Verify that the area $A$ of the polygon satisfies $A = I + B/2 - 1$, where $I$ is the number of lattice points inside the polygon and $B$ is the number of lattice points at the boundary.

2) The integral $\int_0^1 \arccos(\sqrt{x})\ dx$ can be written as a double integral $\int_0^1 \int_0^{\arccos(\sqrt{x})} dydx$. Calculate this integral.