7/29/2004 SECOND HOURLY PRACTICE Maths21a, O. Knill, Summer 2004

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1)	(10 points)

T F The speed of the curve $\vec{r}(t) = (\cos(t), \sin(t), 3t)$ is $(-\sin(t), \cos(t), 3)$.

T F Every smooth function of three variables $f(x,y,z)$ satisfies the partial differential equation $f_{xyz} + f_{yzx} = 2f_{zxy}$.

T F If $f_x(x,y) = f_y(x,y)$ for all x, y, then $f(x,y)$ is a constant.

T F $(1,1)$ is a local maximum of the function $f(x,y) = x^2y - x + \cos(y)$.

T F If f is a smooth function of two variables, then the number of critical points of f inside the unit disc is finite.

T F The value of the function $f(x,y) = \sin(-x+2y)$ at $(0.001, -0.002)$ can by linear approximation be estimated as -0.003.

T F If $(1,1)$ is a critical point for the function $f(x,y)$ then $(1,1)$ is also a critical point for the function $g(x,y) = f(x^2, y^2)$.

T F If the velocity vector $\vec{r}\,'(t)$ of the planar curve $\vec{r}(t)$ is orthogonal to the vector $\vec{r}(t)$ for all times t, then the curve is a circle.

T F The curvature of a circle of radius 10 is $1/10$.

T F The arc length of a curve is given by the formula $\int_a^b \vec{r}'(t)\, dt$.

T F The gradient of $f(x,y)$ is normal to the level curves of f.

T F If (x_0, y_0) is a maximum of $f(x,y)$ under the constraint $g(x,y) = g(x_0, y_0)$, then (x_0, y_0) is a maximum of $g(x,y)$ under the constraint $f(x,y) = f(x_0, y_0)$.

T F If $\vec{u}$ is a unit vector tangent at (x,y,z) to the level surface of $f(x,y,z)$ then the directional derivative satisfies $D_u f(x,y,z) = 0$.

T F If $\vec{r}(t) = (x(t), y(t))$ and $x(t), y(t)$ are polynomials, then the tangent line is defined at all points.

T F The function $u(x,t) = x^2/2 + t$ satisfies the heat equation $u_t = u_{xx}$.

T F The vector $\vec{r}_u(u,v)$ is tangent to the surface parameterized by $\vec{r}(u,v) = (x(u,v), y(u,v), z(u,v))$.

T F Clairots theorem implies $f_{xyx} = f_{yxy}$

T F The second derivative test allows to check whether an extremum found with the Lagrange multiplyer method is a maximum.

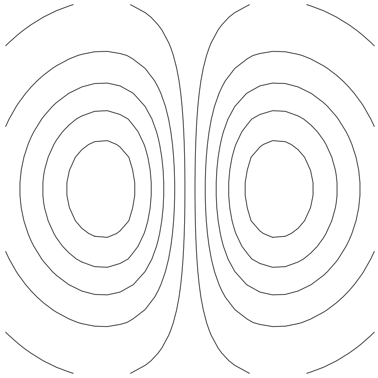
T F If $(0,0)$ is a critical point of $f(x,y)$ and the discriminant D is zero but $f_{xx}(0,0) > 0$ then $(0,0)$ can not be a local maximum.

T F Let (x_0, y_0) be a saddle point of $f(x,y)$. For any unit vector $\vec{u}$, there are points arbitrarily close to (x_0, y_0) for which ∇f is parallel to $\vec{u}$.

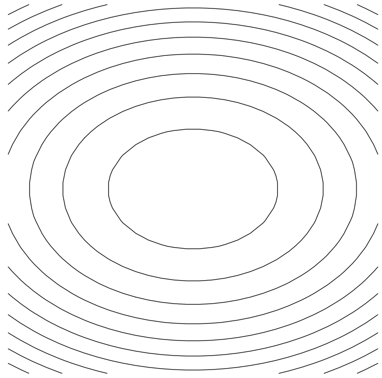
Problem 2) (10 points)

Match the contour maps with the corresponding functions $f(x,y)$ of two variables. Note that one of the contour maps is not represented by a formula. No justifications are needed.

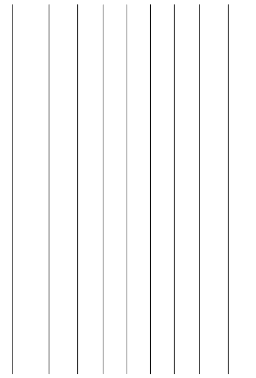
I



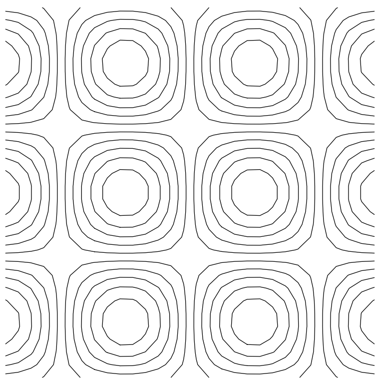
II



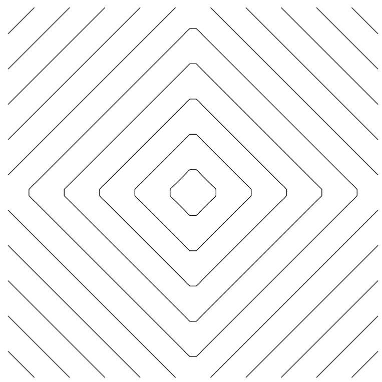
III



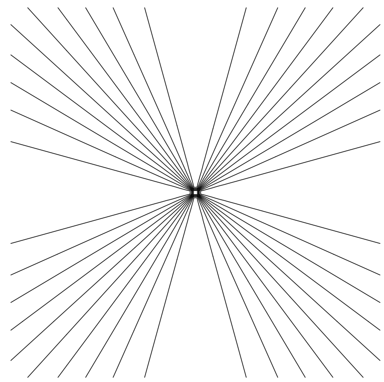
IV



V



VI



Enter I,II,III,IV,V or VI here	Function $f(x,y)$
	$f(x,y) = \sin(x)$
	$f(x,y) = x^2 + 2y^2$
	$f(x,y) = \lvert x\rvert + \lvert y\rvert$
	$f(x,y) = xe^{-x^2-y^2}$
	$f(x,y) = x^2/(x^2+y^2)$

Problem 3) (10 points)

a) Use the technique of linear approximation to estimate $f(\log(2)+0.001, 0.006)$ for $f(x,y) = e^{2x-y}$. (Here, log means the natural logarithm).

b) Find the equation $ax + by = d$ for the tangent line which goes through the point $(\log(2), 0)$.

Problem 4) (10 points)

Find a point on the surface $g(x, y, z) = \frac{1}{x} + \frac{1}{y} + \frac{8}{z} = 1$ for which the distance to the origin is a local minimum.

Problem 5) (10 points)

Find all extrema of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ on the plane and characterize them. Do you find a absolute maximum or absolute minimum amoung them?

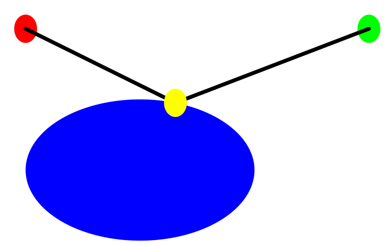
Problem 6) (10 points)

A skydiver feels the acceleration $\vec{r}\,''(t) = (0, 1, -10)$ which is the sum of a gravitational pull $(0, 0, -10)$ and a wind force $(0, 1, 0)$. Assume the skydiver jumps at $t = 0$ from the plane at the position $\vec{r}(0) = (10, 10, 1000)$ and with velocity $\vec{r}\,'(0) = (100, 0, 0)$. Find the position of the skydiver at time $t = 10$.

Problem 7) (10 points)

Find the tangent plane to the surface $f(x, y, z) = x^3 y - xy^2 + 3z = 6$ at the point $(1, 1, 2)$.

Problem 8) (10 points)



You find yourself in the desert at the point $A = (-a, 1)$, completely dehydrated and almost dead. You want to reach the point $B = (b, 1)$ as fast as possible but you can not reach it without water. There is an lake inside the ellipsoid $g(x, y) = x^2 + 2y^2 = 1$. The amount of "effort" you need to go from a point (x, y) to a point (u, v) is assumed to be $(x - u)^2 + (y - v)^2$ (this is justified by the fact that if you walk for a long time, you walk less and less efficiently so that walking twice as long will take you 4 times as much effort). Find the path of least effort which connects A with $X = (x, y)$ and then with B.

a) Which function $f(x, y)$ do you extremize? The parameters a, b are constants.
b) Write down the Lagrange equations.
c) Solve the Lagrange equations in the case $a = -1, b = 1$.

Problem 9) (10 points)

Let $\vec{r}(t)$ be the space curve $\vec{r}(t) = (t^2, \sin(3\pi t), \cos(5\pi t))$.

a) Calculate the velocity, the acceleration and the speed of $\vec{r}(t)$ at time $t = 1$.

b) Write down the integral for the arc length of the curve from $t = 1$ to $t = 10$ as an integral. You don't have to evaluate the integral.
c) The curve $t \mapsto \vec{r}(t) = (t^3, 1 - t, 1 - t^3)$ lies in a plane. What is the equation of this plane?