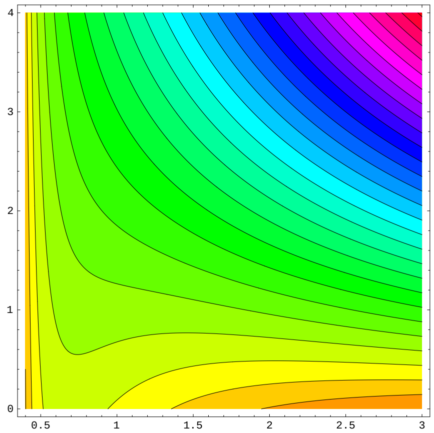
VOLUME-PRESSURE DIAGRAM. When studying gases or liquids, one often describes the process in the **Volume-pressure diagram**. The volume V is plotted on the x-axes and the pressure in the y axes.



Left: V-P diagram
Right: Sadi Carnot

HEAT ENGINE. For periodic processes which happen for example in a fridge, we have a closed cycle given by a closed curve $\gamma : t \mapsto r(t) = (V(t), p(t))$ in the $V - p$ plane. The curve is parameterized by the time t. At a given time, the gas has a specific volume $V(t)$ and a specific pressure $p(t)$.

A LINE INTEGRAL. Consider the vector field $F(V, p) = (p, 0)$ and a closed curve γ and the line integral $\int_\gamma F \, ds$. Writing it out, we get $\int_0^{2\pi} (p(t), 0) \cdot (V'(t), p'(t)) \, dt = \int_0^{2\pi} p(t)V'(t) \, dt = \int_0^{2\pi} p \, dV$.

PHYSICAL INTERPRETATION. If the volume of the gas changes under pressure p, then the work on the system is pdV. On the other hand, if the volume is kept constant, then for a gas, one does not do work on the system, when changing the pressure. Processes described by this approximation are called **adiabatic**.

WORK ON THE GAS. Let us look at a cyclic process, where the volume is decreased under high pressure and increased under low pressure. It is clear that we do some work on the gas. How much is it? We have to compute $\int_0^{2\pi} p \, dV$ for a curve which goes counter clockwise around a closed region R.

PROBLEM. Let $r(t) = (2 + \cos(t), 2 + \sin(t))$ for $t \in [0, 2\pi]$. Compute the line integral of $F(V, p) = (p, 0)$ along that path.

STEPS TO DO THE LINE INTEGRAL.

1) $r'(t) =$

2) $F(r(t)) =$

3) $F(r(t)) \cdot r'(t) =$

4) $\int_0^{2\pi} F(r(t)) \cdot r'(t) \, dt =$