MWF 9 Oliver Knill
MWF 10 Jeremy Hahn
MWF 10 Hunter Spink
MWF 11 Matt Demers
MWF 11 Yu-Wen Hsu
MWF 11 Ben Knudsen
MWF 11 Sander Kupers
MWF 12 Hakim Walker
TTH 10 Ana Balibanu
TTH 10 Morgan Opie
TTH 10 Rosalie Belanger-Rioux
TTH 11:30 Philip Engel
TTH 11:30 Alison Miller

- Start by writing your name in the above box and check your section in the box to the left.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly and except for problems 1-3, give details. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10
13		10
14		10
Total:		150

Problem 1) (20 points) True or False? No justifications are needed.

1) T F Any homogeneous system of linear equations $Ax = 0$ is consistent.

2) T F If P is the matrix of a projection onto a line, then all coefficients of the matrix satisfy $|P_{ij}| \leq 1$.

3) T F The transformation $f(A) = AA^T - 1$ is linear on the 4-dimensional space X of all 2×2 matrices.

4) T F If a smooth 2π periodic function f has a sin-Fourier expansion then its derivative f' has a cos-expansion.

5) T F The characteristic polynomials of two $n \times n$ matrices A, B satisfy $f_A(\lambda) f_B(\lambda) = f_{AB}(\lambda)$.

6) T F The function $f(t) = \cos(10t)$ is an eigenfunction of the linear operator $T = D^4$, where $Df = f'$ is the differentiation operator on $C^\infty(\mathbf{R})$.

7) T F The matrix $A^2 + (A^2)^T$ is diagonalizable, if A is a $n \times n$ matrix.

8) T F The initial value problem $f''(x) + f'(x) + f(x) = \cos(x)$, $f(0) = 0$ has exactly one solution.

9) T F The transformation $T(f)(x) = f(\sin(x))$ is a linear transformation on the space $X = C^\infty(\mathbf{R})$ of smooth functions on the real line.

10) T F The space of smooth functions $f(x,t)$ of two variables which satisfy the partial differential equation $f_{tt} - f_{xx} = f$ is a linear space.

11) T F If A is 6×6 matrix of rank 5, then it has an eigenvalue 0.

12) T F The vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ has the $\mathcal{B} = \{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix} \}$-coordinates $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

13) T F If all the geometric multiplicities of a matrix are equal to the algebraic multiplicities, then the matrix is symmetric.

14) T F All points on the nullclines of a differential equation $\dot{x} = f(x,y), \dot{y} = g(x,y)$ consist of equilibrium points.

15) T F If $z = (a + ib)$ is a nonzero complex number, then $1/z = (a - ib)/(a^2 + b^2)$.

16) T F On the space of 2×2 matrices, the trace tr satisfies $\text{tr}(A^2) = \text{tr}(A)^2$.

17) T F The QR decomposition of an orthogonal matrix A is $A = QR$, where $Q = A$ and $R = 1_n$.

18) T F For any real numbers a, b, there exists a 2×2 matrix such that a is the trace and b is the determinant.

19) T F The dynamical system $x(t+1) = (1/2)x(t) + (1/3)x(t-1)$ has the property that $x(t) \to 0$ for all initial conditions $(x(0), x(1))$.

20) T F For a symmetric matrix A, the kernel of A is perpendicular to the image of A.

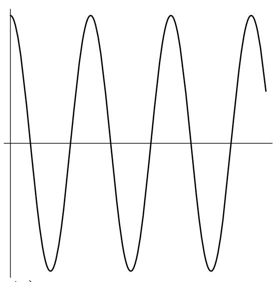
Problem 2) (10 points)

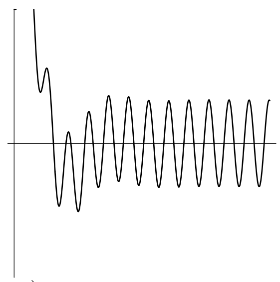
No justifications are needed in this problem. Match the equations with the solution graphs $f(x)$. Enter A,B,C in the right order here:

$f''(x) + f'(x)/5 + f(x) = 0, f(0) = 1, f'(0) = 0$

$f''(x) + f'(x) + f(x) = 5\sin(4x), f(0) = 1, f'(0) = 0$

$f''(x) + f(x) = 0, f(0) = 1, f'(0) = 0.$


A)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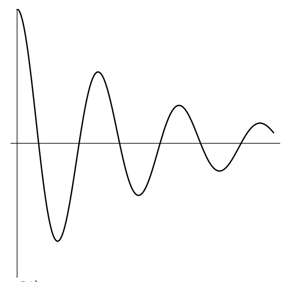
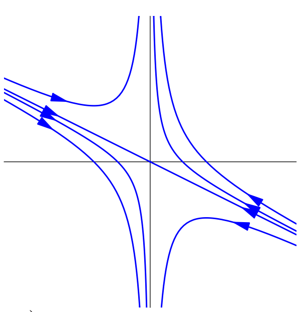

C)

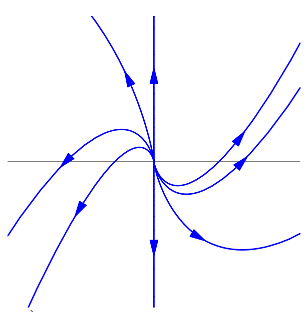
Match the phase portraits for the continuous dynamical systems. Enter D,E,F in the right order here:

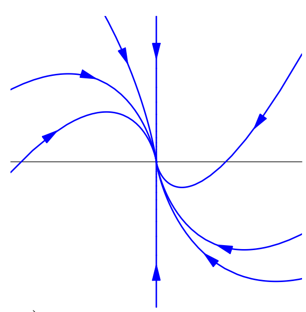
$\frac{d\vec{x}}{dt} = \begin{bmatrix} -2 & 0 \\ -2 & -2 \end{bmatrix} \vec{x}$

$\frac{d\vec{x}}{dt} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} \vec{x}$

$\frac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} \vec{x}$


D)


E)


F)

Problem 3) (10 points)

No justifications are needed in this problem. Fill in a choice of the letters $A - J$ in each of the boxes. No letter will appear twice.

	$\dim(\ker(A)) + \dim(\text{im(A)}) =$ is the number of columns of A.
	If p is a polynomial of degree n, we can write $p(D) = a(D - \lambda_1)(D - \lambda_2)...(D - \lambda_n)$.
	If f is odd, then $\frac{2}{\pi} \int_0^\pi f(x)^2 \, dx = \sum_{n=1}^\infty (\frac{2}{\pi} \int_0^\pi f(x) \sin(nx) \, dx)^2$.
	If $\text{tr}(A) < 0$ and $\det(A) > 0$, then the two dimensional dynamical system is stable.
	The dimension of $\{ f \in C^\infty(\mathbf{R}) \mid p(D)f = 0 \}$ is the degree of the polynomial p.
	If $\int_{-\pi}^\pi f(x)g(x) \, dx = 0$, then $\int_{-\pi}^\pi f(x)^2 \, dx + \int_{-\pi}^\pi g(x)^2 \, dx = \int_{-\pi}^\pi (f(x) + g(x))^2 \, dx$.
	$\cos(\theta) + i \sin(\theta) = e^{i\theta}$.

A) Theorem of Pythagoras for inner products.
B) Fundamental theorem of algebra.
C) Stability for linear continuous dynamical systems in the plane.
D) Percival identity.
E) Rank nullity theorem.
F) Euler's formula.
G) sin-Fourier expansion.
H) Stability criterion for linear discrete dynamical systems in the plane.
I) Parameterization of homogeneous solution space.
J) The Laplace expansion.

Problem 4) (10 points)

Find all the solutions of the system of linear equations:

$$\begin{array}{ccccc} x & +y & +z & +v & = 4 \\ & 2y & +z & +v & = 4 \\ & & 3z & +v & = 4 \\ x & +3y & +2z & +2v & = 8 \end{array}$$

Problem 5) (10 points)

We consider the matrix

$$A = \begin{bmatrix} -6 & 1 & 1 & 1 & 1 & 1 \\ 1 & -6 & 1 & 1 & 1 & 1 \\ 1 & 1 & -6 & 1 & 1 & 1 \\ 1 & 1 & 1 & -6 & 1 & 1 \\ 1 & 1 & 1 & 1 & -6 & 1 \\ 1 & 1 & 1 & 1 & 1 & -6 \end{bmatrix} .$$

a) (2 points) Find all the eigenvalues of $B = A + 7I_6$.
b) (3 points) Find all the eigenvectors of B.
c) (2 points) Find all the eigenvalues of A.
d) (3 points) Decide about the stability for the continuous dynamical system.

$$\dot{x} = Ax .$$

As usual, document all your reasoning.

Problem 6) (10 points)

a) (5 points) Find the 3×3 matrix for the linear transformation which reflects at the plane $x = z$ in three dimensional space.

b) (5 points) What is the 3×3 matrix which rotates by 180 degrees around the line perpendicular to the plane $x = z$ in three dimensional space?

Problem 7) (10 points)

Find the least square solution of the following system of linear equations with the unknown variables x, y:

$$\begin{aligned} x + y &= 1 \\ x - y &= 2 \\ 2x + y &= 1 \end{aligned}$$

Problem 8) (10 points)

The difference equation

$$x_{n+1} = 6x_n - 5x_{n-1}$$

can be written as a discrete dynamical system

$$\begin{bmatrix} x_{n+1} \\ x_n \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_n \\ x_{n-1} \end{bmatrix} .$$

Starting with $x_0 = 0, x_1 = 4$, find an explicit formula for x_n in terms of n.

Problem 9) (10 points)

Find the determinant of the following 36×36 matrix:

$$\begin{bmatrix}
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
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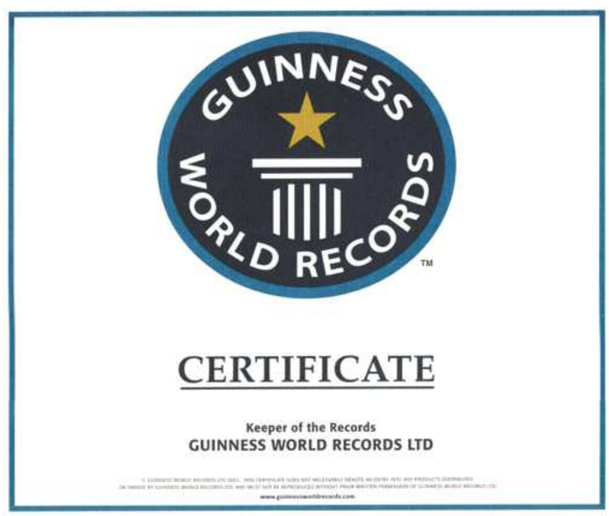
\end{bmatrix} .$$

Make sure you document all your thinking and facts you use about determinants. A detailed protocol can also help to get partial credit.

With this problem we got into the book of Guinness records as the largest matrix which has ever occurred in a linear algebra test.



Problem 10) (10 points)

Find the general solutions for the following differential equations:

a) (2 points) $f'(t) = t$.
b) (2 points) $f'(t) + f(t) = t^2$.
c) (2 points) $f''(t) - f(t) = t$
d) (2 points) $f''(t) + 9f(t) = t$
e) (2 points) $f''(t) - 2f'(t) + f(t) = t$

Problem 11) (10 points)

We analyze the following nonlinear system of differential equations:

$$\begin{aligned} \dot{x} &= y - xy \\ \dot{y} &= x + xy \end{aligned}$$

a) (3 points) Find the nullclines.
b) (3 points) Find all equilibrium points.
c) (4 points) Determine the stability of these equilibrium points.

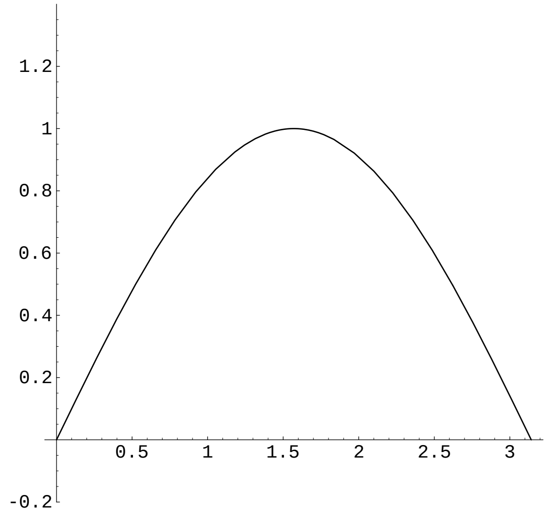
Problem 12) (10 points)

Find the solution of the wave equation $f_{tt} = 16 f_{xx}$ with initial string position

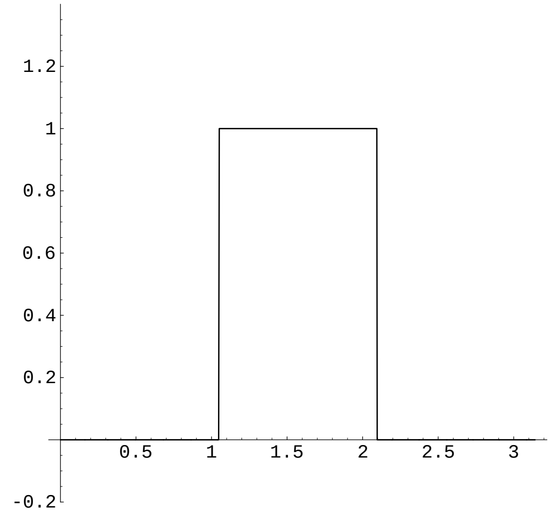
$$f(x,0) = \sin(x)$$

and initial string velocity

$$f_t(x,0) = \begin{cases} 1, & \pi/3 \leq x \leq 2\pi/3 \\ 0, & \text{else} \end{cases} .$$



Initial wave position on $[0, \pi]$.

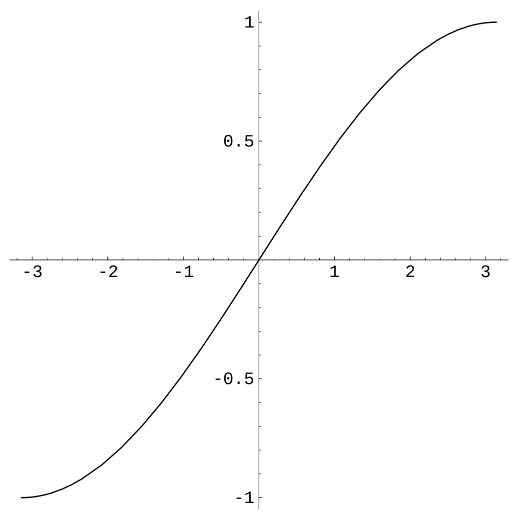


Initial wave velocity on $[0, \pi]$.

Problem 13) (10 points)

a) (6 points) Oliver just bought a new digital piano. One of the wave forms programmed into the piano is the function $\sin(x/2)$ on $[-\pi, \pi]$. Find the Fourier series of this function.

Hint. You can use the formula $2\sin(nx)\sin(mx) = \cos(nx - mx) - \cos(nx + mx)$ and identities like $\sin(a + \pi/2) = \cos(a), \sin(a - \pi/2) = -\cos(a)$.



b) (4 points) What is the value of

$$\sum_{n=1}^{\infty} \frac{n^2}{(1-4n^2)^2} \ ?$$

Problem 14) (10 points)

In this problem, we verify

$$1 + \frac{1}{16} + \frac{1}{64} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90} .$$

It is the value of the famous zeta function $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$ at the point $s = 4$.
We have seen in the handout that $f(x) = x$ has the Fourier expansion

$$x = \sum_{n=1}^{\infty} 2(-1)^{(n+1)} \frac{\sin(nx)}{n} \tag{1}$$

a) You do not have to reverify this here. But we want to know, how the Fourier coefficients $b_n = 2(-1)^{(n+1)}/n$ were obtained in that expansion. Write down the formulas for the Fourier coefficients and especially answer why there are no a_n terms.

b) By taking the anti-derivative of the above formula on both sides, we get

$$g(x) = \frac{x^2}{2} = \sum_{n=1}^{\infty} 2(-1)^n \frac{\cos(nx)}{n^2} + a_0 \frac{1}{\sqrt{2}} .$$

Find a_0, the zero'th Fourier coefficient of the function $\frac{x^2}{2}$.

c) Compute the length $||g||^2 = \langle g, g \rangle$ and deduce from this

$$4 \sum_{n=1}^{\infty} \frac{1}{n^4} = (\frac{\pi^4}{10} - \frac{\pi^4}{18}) = 2\pi^4/45$$

which implies $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$.