The rule of three

One of the oldest mathematical tasks is the problem to solve one single linear equation. One of the results there is the **rule of three** or **cross multiplication**: In the intro meeting, I mentioned the problem Suppose $1+1/2$ cats eat $1+1/2$ mice in $1+1/2$ hours. Then 4 cats eat 4 mice in how many hours? We have to realize that more cats need less time and more mice needs more time. The two factors are both linear. We have 4 cats eating 4 mice in the same time, in $1+1/2$ hours. The next step is including addition of constants:

$$ax + b = cx + d \ .$$

To solve this, we move all variables one one side, $(a-c)x = d-b$ then solve for $x = (d-b)/(a-c)$.

Egypt: Rhind Papyrus

Here is a historical problem:

> **Divide 100 loaves among 10 men including a boatman, a foreman and a doorkeeper who receive double portions. What is the share of each?**

It requires to solve $7x + 3 * (2x) = 100$ for x. The solution is x=200/13 for the three special ones and 100/13 for the rest.

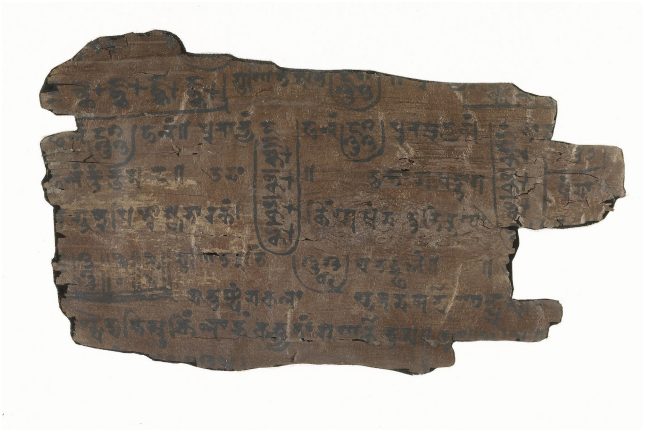
Babylonians: Tell Harmal excavations

If somebody asks you thus:
if I add to the two thirds
of my two thirds a hundred aq
of barley, the original quantity is
summed up. How much is the original quantity?

$$(4/9)x + 100 = x$$

Source: George Gheverghese Joseph: The crest of the peacock: Non-European roots of Mathematics

India: Bakshali manuscript



Now, we come to systems of equations. One of the first places, where systems of equations appeared was in China and India. Here is an example from the Bakshali manuscript:

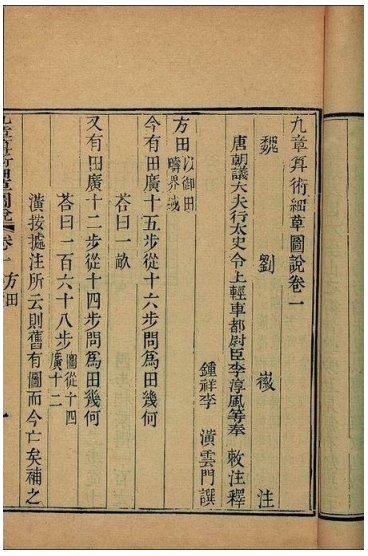
One person possesses seven Asava horses,
another nine Haya horses, and another ten camels.
Each gives two animals, one to each of the others.
They are then equally well off. Find the price of
each animal and the total value of the animals
possesses by each person.

x asava, y haya and z camels, t total

$$\begin{aligned} 5x + y + z &= t \\ 7y + x + z &= t \\ 8z + y + x &= t \end{aligned}$$

China: Nine Chapters of Mathematical Art

The nine chapters of mathematical art seem definitely one of the first text sources, which treated systems of linear equations. Here is an example:



$$\begin{aligned} 3x + 2y + z &= 39 \\ 2x + 3y + z &= 34 \\ x + 2y + 3z &= 26 \end{aligned}$$

Han Dynasty, 200 BC

Historical Moments

Here are some pointers

2000 BC: Babylonians: one variable
2000 BC: Egypt: Rhind papyrus one variable
200BC-200CE: China: Jiuzhang Suanshu: Nine chapters of mathematical art.
300: India: Bakshali manuscript
829: AlKhawarizmi: systematic way to solve linear and quadratic equations
1575: Viete: use of variables.
1678: Leibniz: Determinant definition 1750: Cramer: rule using determinants
1809: Gauss: Motion of heavily bodies, elimination and least square solutions
1850: Cayley: algebra of matrices