7.1.12 Solving $\begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 2 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ we get $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} t \\ -\frac{3}{2}t \end{bmatrix}$ (with $t \neq 0$) and

solving $\begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 4 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ we get $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ t \end{bmatrix}$ (with $t \neq 0$).

7.1.36 We want A such that $A \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$ and $A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$, i.e. $A \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 10 \\ 5 & 20 \end{bmatrix}$, so

$$A = \begin{bmatrix} 15 & 10 \\ 5 & 20 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{1} = \begin{bmatrix} 4 & 3 \\ -2 & 11 \end{bmatrix}.$$

7.2.12 $f_A(\lambda) = \lambda(\lambda+1)(\lambda-1)^2$ so $\lambda_1 = 0$, $\lambda_2 = -1$, $\lambda_3 = 1$ (Algebraic multiplicity 2).

7.2.28 a $w(t+1) = 0.8w(t) + 0.1m(t)$

$m(t+1) = 0.2w(t) + 0.9m(t)$

so $A = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$ which is a regular transition matrix since its columns sum to 1 and its entries are positive.

b The eigenvectors of A are $\begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ with $\lambda_1 = 1$, and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ with $\lambda_2 = 0.7$.

$\vec{x}_0 = \begin{bmatrix} 1200 \\ 0 \end{bmatrix} = 400 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 800 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ so $\vec{x}(t) = 400 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 800(0.7)^t \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ or

$w(t) = 400 + 800(0.7)^t$

$m(t) = 800 - 800(0.7)^t$.

c As $t \to \infty$, $w(t) \to 400$ so Wipfs won't have to close the store.

7.2.38 By Theorem 7.2.4, the characteristic polynomial of A is $f_A(\lambda) = \lambda^2 - 5\lambda - 14 = (\lambda - 7)(\lambda + 2)$, so that the eigenvalues are 7 and -2.

7.2.25 $A\begin{bmatrix} b \\ c \end{bmatrix} = \begin{bmatrix} ab+cb \\ cb+cd \end{bmatrix} = \begin{bmatrix} (a+c)b \\ (b+d)c \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix}$ since $a+c=b+d=1$; therefore, $\begin{bmatrix} b \\ c \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda_1 = 1$.

Also, $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} a-b \\ c-d \end{bmatrix} = (a-b)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ since $a-b=-(c-d)$; therefore, $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda_2 = a-b$. Note that $|a-b|<1$; a possible phase portrait is shown in Figure 7.11.

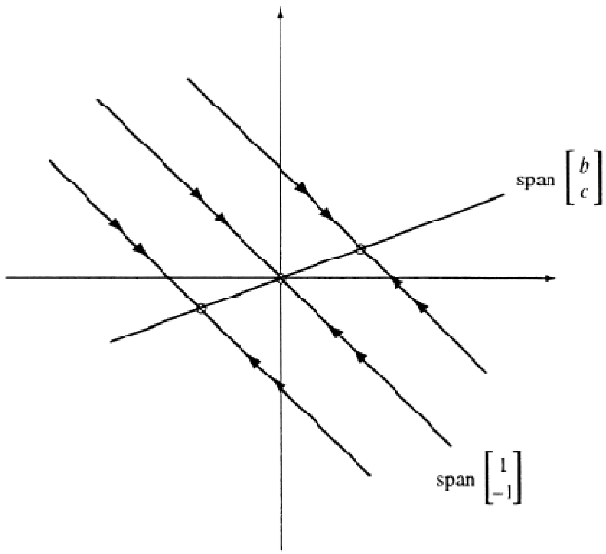

Figure 7.116: for Problem 7.2.25.

7.2.26 Here $\begin{bmatrix} b \\ c \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix}$ with $\lambda_1 = 1$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ with $\lambda_2 = a - b = 0.25$. See Figure 7.12.

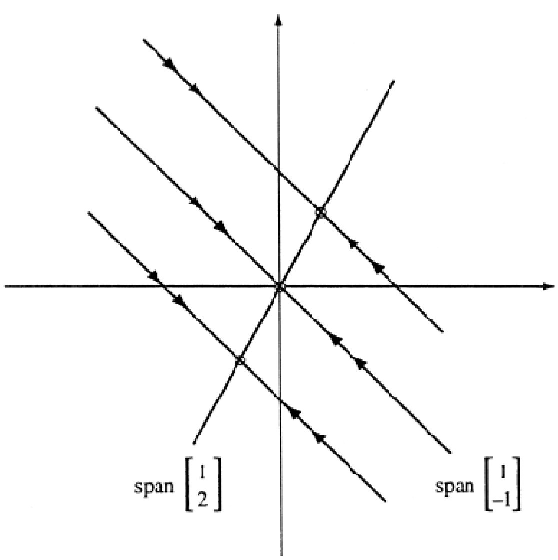

Figure 7.117: for Problem 7.2.26.