5.3.5 $3A$ will not be orthogonal, because the length of the column vectors will be 3 instead of 1, and they will fail to be unit vectors.

5.3.6 $-B$ will certainly be orthogonal, since the columns will be perpendicular unit vectors.

5.3.7 AB is orthogonal by Theorem 5.3.4a.

5.3.8 $A+B$ will not necessarily be orthogonal, because the columns may not be unit vectors. For example, if $A = B = I_n$, then $A + B = 2I_n$, which is not orthogonal.

5.3.9 B^{-1} is orthogonal by Theorem 5.3.4b.

5.3.10 This matrix will be orthogonal, by Theorem 5.3.4.

5.3.11 A^T is orthogonal. $A^T = A^{-1}$, by Theorem 5.3.7, and A^{-1} is orthogonal by Theorem 5.3.4b.

5.3.13 $3A$ is symmetric, since $(3A)^T = 3A^T = 3A$.

5.3.14 $-B$ is symmetric, since $(-B)^T = -B^T = -B$.

5.3.15 AB is not necessarily symmetric, since $(AB)^T = B^TA^T = BA$, which is not necessarily the same as AB. (Here we used Theorem 5.3.9a.)

5.3.16 $A + B$ is symmetric, since $(A + B)^T = A^T + B^T = A + B$.

5.3.17 B^{-1} is symmetric, because $(B^{-1})^T = (B^T)^{-1} = B^{-1}$. In the first step we have used 5.3.9b.

5.3.18 A^{10} is symmetric, since $(A^{10})^T = (A^T)^{10} = A^{10}$.

5.3.19 This matrix is symmetric. First note that $(A^2)^T = (A^T)^2 = A^2$ for a symmetric matrix A. Now we can use the linearity of the transpose, $(2I_n + 3A - 4A^2)^T = 2I_n^T + 3A^T - (4A^2)^T = 2I_n + 3A - 4(A^T)^2 = 2I_n + 3A - 4A^2$.

5.3.20 AB^2A is symmetric, since $(AB^2A)^T = (ABBA)^T = (BA)^T(AB)^T = A^TB^TB^TA^T = AB^2A$.

5.3.40 An orthonormal basis of W is $\vec{u}_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} -0.1 \\ 0.7 \\ -0.7 \\ 0.1 \end{bmatrix}$ (see Exercise 5.2.9).

By Theorem 5.3.10, the matrix of the projection onto W is QQ^T, where $Q = [\,\vec{u}_1 \quad \vec{u}_2\,]$.

$$QQ^T = \frac{1}{100}\begin{bmatrix} 26 & 18 & 32 & 24 \\ 18 & 74 & -24 & 32 \\ 32 & -24 & 74 & 18 \\ 24 & 32 & 18 & 26 \end{bmatrix}$$

5.3.48 As suggested, we consider the QR factorization

$A^T = PR$

of A^T, where P is orthogonal and R is upper triangular with positive diagonal entries.

By Theorem 5.3.9a, $A = (PR)^T = R^T P^T$.

Note that $L = R^T$ is lower triangular and $Q = P^T$ is orthogonal.

5.3.46 By Theorem 5.2.2, the columns $\vec{u}_1, \ldots, \vec{u}_m$ of Q are orthonormal. Therefore, $Q^T Q = I_m$, since the ijth entry of $Q^T Q$ is $\vec{u}_i \cdot \vec{u}_j$.

If we multiply the equation $M = QR$ by Q^T from the left then $Q^T M = Q^T QR = R$, as claimed.

Ch **5.TF.22** T, by Theorem 5.4.1.

Ch **5.TF.23** T, by Theorem 5.4.2a.