2.1.6 Note that $x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, so that T is indeed linear, with matrix

$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$.

2.1.14 a By Exercise 13a, $\begin{bmatrix} 2 & 3 \\ 5 & k \end{bmatrix}$ is invertible if (and only if) $2k - 15 \neq 0$, or $k \neq 7.5$.

b By Exercise 13b, $\begin{bmatrix} 2 & 3 \\ 5 & k \end{bmatrix}^{-1} = \frac{1}{2k-15} \begin{bmatrix} k & -3 \\ -5 & 2 \end{bmatrix}$.

If all entries of this inverse are integers, then $\frac{3}{2k-15} - \frac{2}{2k-15} = \frac{1}{2k-15}$ is a (nonzero) integer n, so that $2k - 15 = \frac{1}{n}$ or $k = 7.5 + \frac{1}{2n}$. Since $\frac{k}{2k-15} = kn = 7.5n + \frac{1}{2}$ is an integer as well, n must be odd.

We have shown: If all entries of the inverse are integers, then $k = 7.5 + \frac{1}{2n}$, where n is an odd integer. The converse is true as well: If k is chosen in this way, then the entries of $\begin{bmatrix} 2 & 3 \\ 5 & k \end{bmatrix}^{-1}$ will be integers.

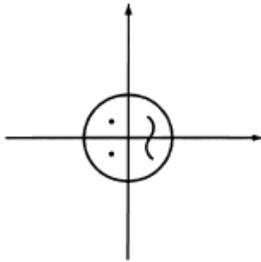**2.1.24** Compare with Example 5. (See Figure 2.5.)

Figure 2.21: for Problem 2.1.24.

2.1.25 The matrix represents a scaling by the factor of 2. (See Figure 2.6.)

2.1.26 This matrix represents a reflection about the line $x_2 = x_1$. (See Figure 2.7.)
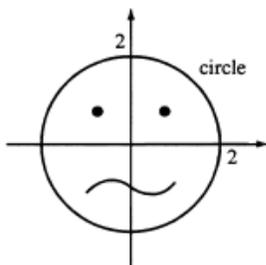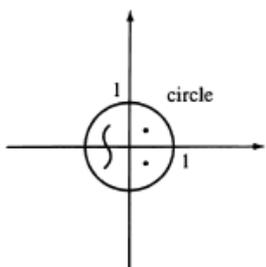
Figure 2.22: for Problem 2.1.25.

Figure 2.23: for Problem 2.1.26.

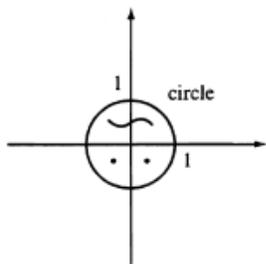**2.1.27** This matrix represents a reflection about the $\vec{e}_1$ axis. (See Figure 2.8.)

Figure 2.24: for Problem 2.1.27.

2.1.28 If $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, then $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 2x_2 \end{bmatrix}$, so that the x_2 component is multiplied by 2, while the x_1 component remains unchanged. (See Figure 2.9.)

2.1.29 This matrix represents a reflection about the origin. Compare with Exercise 17. (See
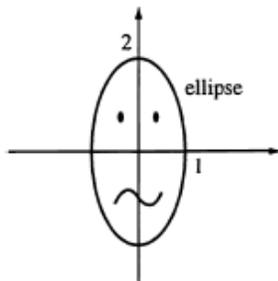
Figure 2.25: for Problem 2.1.28.
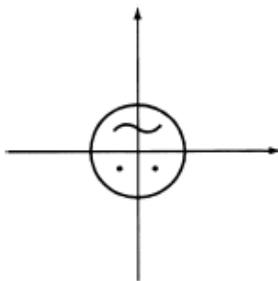
Figure 2.10.)

Figure 2.26: for Problem 2.1.29.

2.1.30 If $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, then $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ x_2 \end{bmatrix}$, so that A represents the projection onto the $\vec{e}_2$ axis. (See Figure 2.11.)
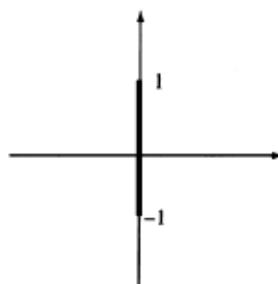

Figure 2.27: for Problem 2.1.30.

2.1.42 a See Figure 2.16.

b The image of the point $\begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$ is the origin, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.
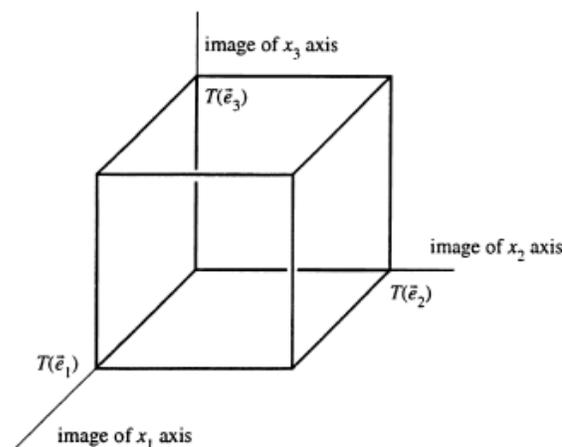

Figure 2.32: for Problem 2.1.42.

c Solve the equation $\begin{bmatrix} -\frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, or $\begin{bmatrix} -\frac{1}{2}x_1 & + & x_2 & & = 0 \\ -\frac{1}{2}x_1 & + & & x_3 & = 0 \end{bmatrix}$. (See Figure 2.16.)

The solutions are of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2t \\ t \\ t \end{bmatrix}$, where t is an arbitrary real number. For example, for $t = \frac{1}{2}$, we find the point $\begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$ considered in part b. These points are on the line through the origin and the observer's eye.

2.1.44 $T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} v_2x_3 - v_3x_2 \\ v_3x_1 - v_1x_3 \\ v_1x_2 - v_2x_1 \end{bmatrix} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, so that

T is linear, with matrix $\begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$.

2.1.34 As in Exercise 2.1.33, we find $T(\vec{e}_1)$ and $T(\vec{e}_2)$; then by Theorem 2.1.2, $A = [T(\vec{e}_1)\, T(\vec{e}_2)]$. (See Figure 2.13.)

Therefore, $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

2.1.52 a $A\vec{x} = \begin{bmatrix} 300 \\ 2400 \end{bmatrix}$, meaning that the total value of our money is C\$300, or, equivalently $ZAR2400$.

b From Exercise 13, we test the value $ad - bc$ and find it to be zero. Thus A is not invertible. To determine when A is consistent, we begin to compute rref $\left[A \vdots \vec{b} \right]$:

$$\begin{bmatrix} 1 & \frac{1}{8} & \vdots & b_1 \\ 8 & 1 & \vdots & b_2 \end{bmatrix} \xrightarrow[-8I]{} \begin{bmatrix} 1 & \frac{1}{8} & \vdots & b_1 \\ 0 & 0 & \vdots & b_2 - 8b_1 \end{bmatrix}.$$

Thus, the system is consistent only when $b_2 = 8b_1$. This makes sense, since b_2 is the total value of our money in terms of Rand, while b_1 is the value in terms of Canadian dollars. Consider the example in part a. If the system $A\vec{x} = \vec{b}$ is consistent, then there will be

infinitely many solutions $\vec{x}$, representing various compositions of our portfolio in terms of Rand and Canadian dollars, all representing the same total value.

Ch 2.TF.5 F; Matrix AB will be 3×5, by Definition 2.3.1b.

Ch 2.TF.6 F; Note that $T \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. A linear transformation transforms $\vec{0}$ into $\vec{0}$.