1.2.6 The system is in rref already.

$$\begin{bmatrix} x_1 & = 3 + 7x_2 - x_5 \\ x_3 & = 2 + 2x_5 \\ x_4 & = 1 - x_5 \end{bmatrix}$$

Let $x_2 = t$ and $x_5 = r$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 + 7t - r \\ t \\ 2 + 2r \\ 1 - r \\ r \end{bmatrix}$$

1.2.12 The system reduces to $\begin{bmatrix} x_1 & & & & + & 3.5x_5 & + & x_6 & = & 0 \\ & x_2 & & & + & x_5 & & & = & 0 \\ & & x_3 & & & & - & \frac{5}{3}x_6 & = & 0 \\ & & & x_4 & + & 3x_5 & + & x_6 & = & 0 \end{bmatrix} \longrightarrow$

$$\begin{bmatrix} x_1 & = -3.5x_5 - x_6 \\ x_2 & = -x_5 \\ x_3 & = \frac{5}{3}x_6 \\ x_4 & = -3x_5 - x_6 \end{bmatrix}.$$

Let $x_5 = r$ and $x_6 = t$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -3.5r - t \\ -r \\ \frac{5}{3}t \\ -3r - t \\ r \\ t \end{bmatrix}$$

1.2.18 a No, since the third column contains two leading ones.

b Yes

c No, since the third row contains a leading one, but the second row does not.

d Yes

1.2.30 Plugging the points into $f(t)$, we obtain the system

$$\begin{bmatrix} a & & & & & & & = & 1 \\ a & + & b & + & c & + & d & = & 0 \\ a & - & b & + & c & - & d & = & 0 \\ a & + & 2b & + & 4c & + & 8d & = & -15 \end{bmatrix}$$

with unique solution $a = 1$, $b = 2$, $c = -1$, and $d = -2$, so that $f(t) = 1 + 2t - t^2 - 2t^3$. (See Figure 1.8.)

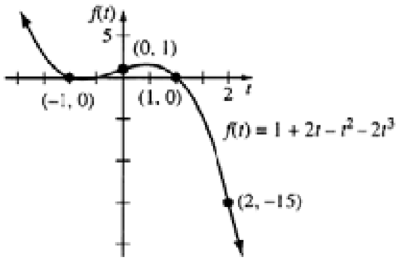

Figure 1.8: for Problem 1.2.30.

1.2.42 Let x_1, x_2, x_3, and x_4 be the traffic volume at the four locations indicated in Figure 1.11.

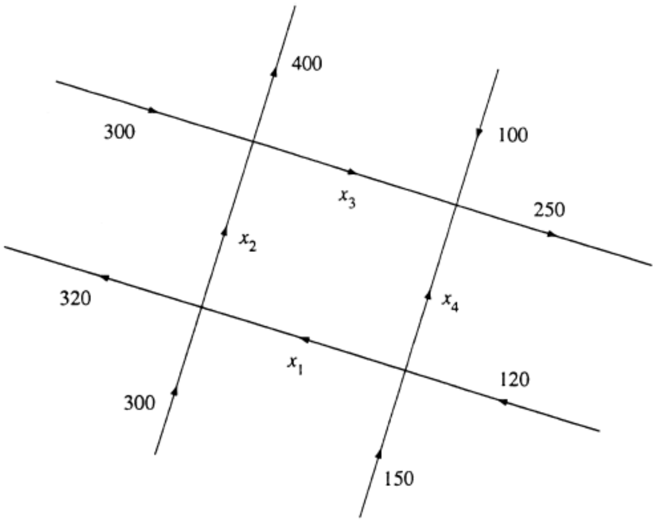

Figure 1.11: for Problem 1.2.42.

We are told that the number of cars coming into each intersection is the same as the number of cars coming out:

$$\begin{bmatrix} x_1+300 & =320+x_2 \\ x_2+300 & =400+x_3 \\ x_3+x_4+100 & =250 \\ 150+120 & =x_1+x_4 \end{bmatrix} \text{ or } \begin{bmatrix} x_1 & - & x_2 & & & & & = & 20 \\ & & x_2 & - & x_3 & & & = & 100 \\ & & & & x_3 & + & x_4 & = & 150 \\ x_1 & & & & & + & x_4 & = & 270 \end{bmatrix}$$

The solutions are of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 270-t \\ 250-t \\ 150-t \\ t \end{bmatrix}$.

Since the x_i must be positive integers (or zero), t must be an integer with $0 \leq t \leq 150$.

The lowest possible values are $x_1 = 120,\ x_2 = 100,\ x_3 = 0$, and $x_4 = 0$, while the highest possible values are $x_1 = 270,\ x_2 = 250,\ x_3 = 150$, and $x_4 = 150$.

1.2.32 The requirement $f_i'(a_i) = f_{i+1}'(a_i)$ and $f_i''(a_i) = f_{i+1}''(a_i)$ ensure that at each junction two different cubics fit "into" one another in a "smooth" way, since they must have the same slope and be equally curved. The requirement that $f_1'(a_0) = f_n'(a_n) = 0$ ensures that the track is horizontal at the beginning and at the end. How many unknowns are there? There are n pieces to be fit, and each one is a cubic of the form $f(t) = p+qt+rt^2+st^3$, with p, q, r, and s to be determined; therefore, there are $4n$ unknowns. How many equations are there?

$f_i(a_i) = b_i$	for $i = 1, 2, \ldots, n$	gives n equations
$f_i(a_{i-1}) = b_{i-1}$	for $i = 1, 2, \ldots, n$	gives n equations
$f_i'(a_i) = f_{i+1}'(a_i)$	for $i = 1, 2, \ldots, n-1$	gives $n-1$ equations
$f_i''(a_i) = f_{i+1}''(a_i)$	for $i = 1, 2, \ldots, n-1$	gives $n-1$ equations
$f_1'(a_0) = 0, f_n'(a_n) = 0$		gives 2 equations

Altogether, we have $4n$ equations; convince yourself that all these equations are linear.

1.2.20 Four, namely $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & k \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (k is an arbitrary constant.)

Ch 1.TF.**6** F; As a counter-example, consider the zero matrix.

Ch 1.TF.**44** F; Consider $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$. If we remove the first column, then the remaining matrix fails to be in rref.