9.2.12 We will show that the real parts of all the eigenvalues are negative, so that the zero state is a stable equilibrium solution. Now the characteristic polynomial of A is $f_A(\lambda) = -\lambda^3 - 2\lambda^2 - \lambda - 1$. It is convenient to get rid of all these minus signs: The eigenvalues are the solutions of the equation $g(\lambda) = \lambda^3 + 2\lambda^2 + \lambda + 1 = 0$. Since $g(-1) = 1$ and $g(-2) = -1$, there will be an eigenvalue λ_1 between -2 and -1. Using calculus (or a graphing calculator), we see that the equation $g(\lambda) = 0$ has no other real solutions. Thus there must be two complex conjugate eigenvalues $p \pm iq$. Now the sum of the eigenvalues is $\lambda_1 + 2p = \text{tr}(A) = -2$, and $p = \frac{-2-\lambda_1}{2}$ will be negative , as claimed. The graph of $g(\lambda)$ is shown in Figure 9.33.

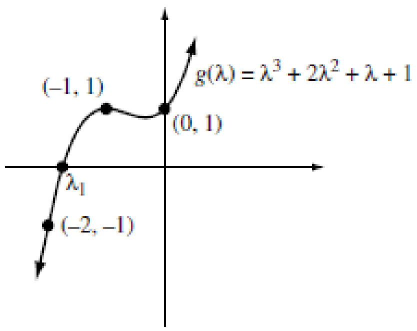

Figure 9.194: for Problem 9.2.12.

9.2.18 If λ_1, λ_2, λ_3 are real and negative, then $\text{tr}(A) = \lambda_1 + \lambda_2 + \lambda_3 < 0$ and $\det(A) = \lambda_1\lambda_2\lambda_3 < 0$. If λ_1 is real and negative and $\lambda_{2,3} = p \pm iq$, where p is negative, then $\text{tr}(A) = \lambda_1 + 2p < 0$ and $\det(A) = \lambda_1(p^2 + q^2) < 0$. Either way, both trace and determinant are negative.

9.2.22 $\lambda_1 = 3, \lambda_2 = 0.5; E_3 = \text{span}\begin{bmatrix} 1 \\ -1 \end{bmatrix}, E_{0.5} = \text{span}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

System is discrete so choose VII.

9.2.23 $\lambda_{1,2} = -\frac{1}{2} \pm i, r > 1$, so that trajectory spirals outwards. Choose II.

9.2.24 $\lambda_1 = 3, \lambda_2 = 0.5, E_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, E_{0.5} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

System is continuous, so choose I.

9.2.25 $\lambda_{1,2} = -\frac{1}{2} \pm i$; real part is negative so that trajectories spiral inwards in the counterclockwise direction $\left(\text{if } \vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ then } \frac{d\vec{x}}{dt} = \begin{bmatrix} -1.5 \\ 2 \end{bmatrix}\right)$. Choose IV.

9.2.26 $\lambda_1 = 1, \lambda_2 = -2; E_1 = \text{span}\begin{bmatrix} 0 \\ 1 \end{bmatrix}, E_{-2} = \text{span}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

System is continuous so choose V.

9.2.34 $\lambda_{1,2} = 1 \pm 2i$, $E_{1+2i} = \text{span}\left(\begin{bmatrix} 3 \\ -2 \end{bmatrix} + i \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$

$a = 1, b = 0$, so that $\vec{x}(t) = e^t \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} \cos(2t) & -\sin(2t) \\ \sin(2t) & \cos(2t) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = e^t \begin{bmatrix} \cos(2t) + 3\sin(2t) \\ -2\sin(2t) \end{bmatrix}$.
See Figure 9.39.

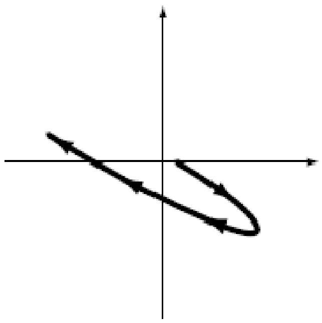

Figure 9.200: for Problem 9.2.34.

9.2.40 a $B(t) = 1000(1 + 0.05i)^t = 1000(r(\cos\theta + i\sin\theta))^t = 1000r^t(\cos(\theta t) + i\sin(\theta t))$, where

$r = \sqrt{1 + 0.05^2} > 1$ and $\theta = \arctan(0.05) \approx 0.05$. See Figure 9.42.

b $B(t) = 1000e^{0.05i} = 1000(\cos(0.05t) + i\sin(0.05t))$. See Figure 9.42.

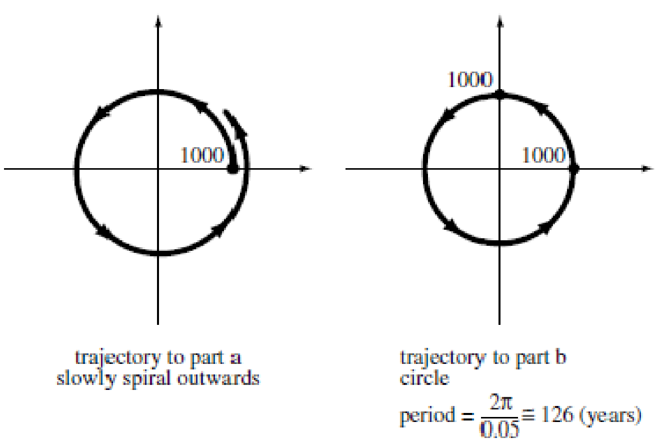

Figure 9.203: for Problem 9.2.40.

c We would choose an account with annual compounding, since the modulus of the balance grows in this case. In the case of continuous compounding the modulus of the balance remains unchanged.

9.2.36 a If $c = 0$ then $\lambda_{1,2} = \pm i\sqrt{b}$. The trajectories are ellipses. See Figure 9.40.

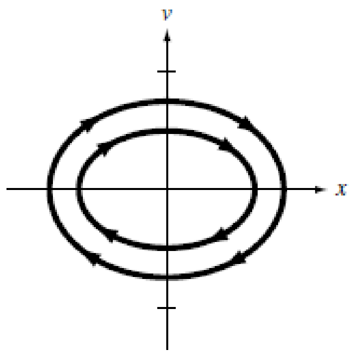

Figure 9.201: for Problem 9.2.36a.

The block *oscillates harmonically*, with period $\frac{2\pi}{\sqrt{b}}$. The zero state fails to be asymptotically stable.

b $\lambda_{1,2} = \frac{-c \pm i\sqrt{4b - c^2}}{2}$

The trajectories spiral inwards, since $\text{Re}(\lambda_1) = \text{Re}(\lambda_2) = -\frac{c}{2} < 0$. This is the case of a *damped oscillation*. The zero state is asymptotically stable. See Figure 9.41.

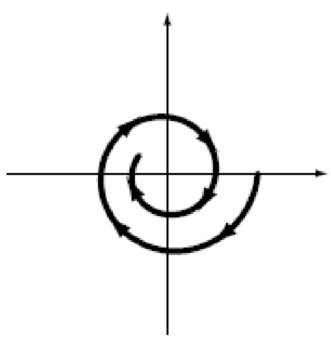

Figure 9.202: for Problem 9.2.36b.

c This case is discussed in Exercise 9.1.55. The zero state is stable here.