ORGANISATION OF THIS SECTION:

- Instructor: Oliver Knill
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- Office: 434, Science Center
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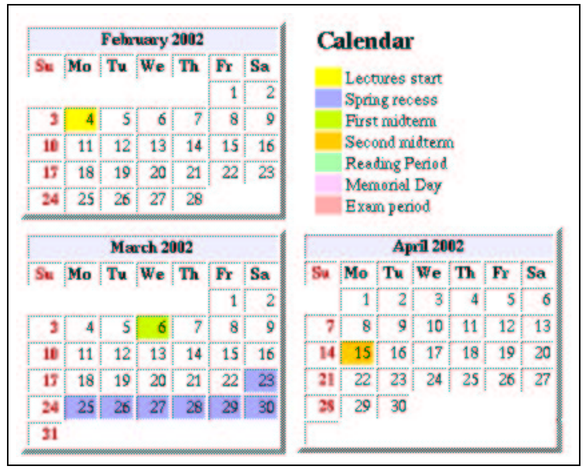
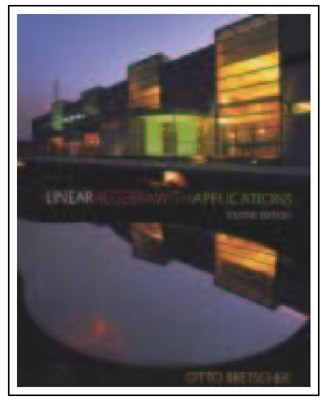
- CA: Jonathan Lerner, Tel: 3-6034, e-mail lerner@fas.harvard.edu
- Course page: http://www.courses.fas.harvard.edu/˜math21b/
- Course head: Richard Taylor, rtaylor@math, Office 504
- Midterms Wed, Mar 6, 7:00-8:45pm in Sci Ctr Hall D, Mon, Apr 15, 7:00-8:45pm in Sci Ctr Hall C
- Textbook: Linear Algebra and its applications by Otto Bretscher (second edition)
- Grade: Midterms 20% each, homework: 20 %, Final: 40 %.
- Homework: Due at start of each class.





1.1: 10,14,20*,24,26,36* **Due:** Thu 2/7/2002
1.2: 6,11,18,20,30,38*, 1.3:4,14,26*,34,48,50, **Due:** Tue 2/12/2002

LINEAR EQUATIONS. A collection of linear equations is called a **system of linear equations**. An example is

$$\left| \begin{array}{ll} 3x - y - z & = 0 \\ -x + 2y - z & = 0 \\ -x - y + 3z & = 9 \end{array} \right|$$

There are three equations. The unknows are x, y, z. **Linear** means that no nonlinear terms like $x^2, x^3, xy, yz^3, \sin(x)$ etc. appear.

SOLVING THE EQUATIONS I).
Eliminate variables. In the example the first equation gives $z = 3x - y$. Putting this into the second and third equation gives

$$\left| \begin{array}{ll} -x + 2y - (3x - y) & = 0 \\ -x - y + 3(3x - y) & = 9 \end{array} \right|$$

or

$$\left| \begin{array}{ll} -4x + 3y & = 0 \\ 8x - 4y & = 9 \end{array} \right| .$$

The first equation gives $y = 4/3x$ and plugging this into the other equation gives $8x - 16/3x = 9$ or $8x = 27$ which means $x = 27/8$. The other values $y = 9/2, z = 45/8$ can now be obtained.

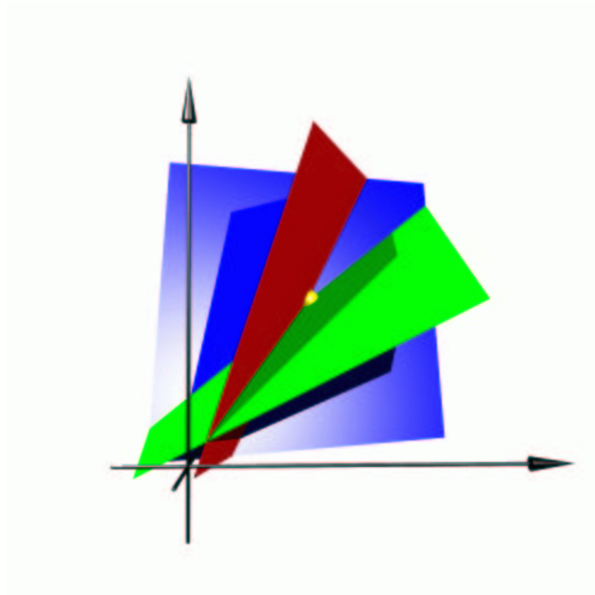
SOLVING THE EQUATIONS II).
Addition of equations. If we subtract the third equation from the second, we get $3y - 4z = -9$ and add three times the second equation to the first, we get $5y - 4z = 0$. Subtracting this equation to the previous one gives $-2y = -9$ or $y = 2/9$.

SOLVING THE EQUATIONS III).
Use the computer. In Mathematica:

$$\text{Solve}[\{3x - y - z == 0, -x + 2y - z == 0, -x - y + 3z == 9\}, \{x, y, z\}] \;.$$

But what did Mathematica do to solve this equation? We will look in this course at some efficient algorithms.

SOLVING THE EQUATIONS IV).
Geometric solution. Each equation represents a plane in three-dimensional space. To satisfy the first equation is equivalent with beeing on the first plane. To satisfy the second equation means to be on the second plane. To satisfy the first two equations means to be on the intersection line of these two planes. To satisfy all three equations, we have to intersect the line with the plane representing the third equation.

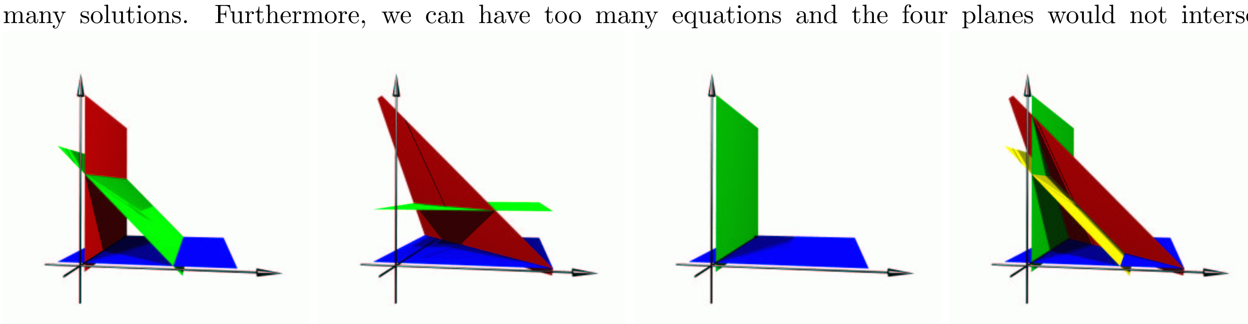


MURPHYS LAW.

"If anything can go wrong, it will go wrong".

"If you are feeling good, don't worry, you will get over it!"

MURPHYS LAW IS TRUE. Two equations could contradict each other. Geometrically this means that the two planes don't intersect. This is possible if they are parallel. Even without two planes being parallel, it is possible that there is no intersection between all three of them. Also possible is that we don't have enough equations (for example because two equations are the same) and that there are many solutions. Furthermore, we can have too many equations and the four planes would not intersect.



RELEVANCE OF EXCEPTIONAL CASES. There are important applications, where "unusual" sitations happen: For example in medical tomography, systems of equations appear which are "ill posed". In this case one has to be careful with the method. The linear equations are then obtained from a method called the **Radon transform**. The task for finding a good method had led to a Nobel prize in Medicin 1979 (Allan Cormack). Cormack had sabbatticals at Harvard and probably has done part of his work on tomography here. Tomography helps today for example for cancer treatement.