Monday no class: homework for Wednesday, Februrary 21: Section 2.3: 10,20,26*,30,40,42*

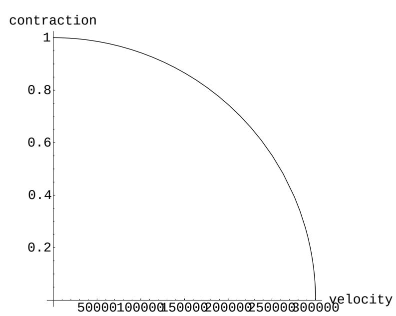
The Lorentz Boost 2/15/2001 — Oliver Knill

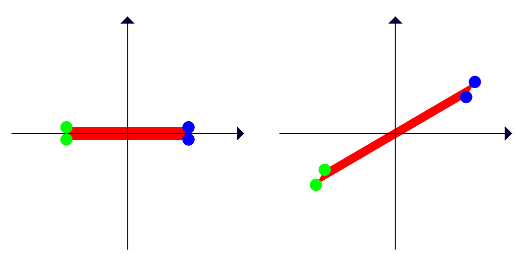
(This is background information to one of the examples last time, it is not part of this course. The physical interpretation in special relativity might be fun to know about it.)

The Lorentz boost. The linear transformation of the plane given by the matrix $A = \begin{pmatrix} \cosh(\phi) & \sinh(\phi) \\ \sinh(\phi) & \cosh(\phi) \end{pmatrix}$ is called the **Lorentz boost**. The transformation $\begin{pmatrix} x \\ y \end{pmatrix} \mapsto A \begin{pmatrix} x \\ y \end{pmatrix}$ with $y = ct$ has a physical interpretation.

Physical interpretation. When a particle travels with a velocity v on the line, its new position satisfies $\tilde{x} = x + tv$. According to special relativity, this is only an approximation. In reality, the motion is described by $\begin{pmatrix} x \\ ct \end{pmatrix} \mapsto A \begin{pmatrix} x \\ ct \end{pmatrix}$ The angle ϕ is related to the velocity v by the formula $\tanh(\phi) = v/c$. Trigonometric identities give $\sinh(\phi) = (v/c)/\gamma, \cosh(\phi) = 1/\gamma$, where $\gamma = \sqrt{1 - v^2/c^2}$. The linear transformation tells then $A(x, ct) = ((x + vt)/\gamma, t + (v/c^2)/\gamma x)$. For small velocities v, the value of γ is close to 1 and v/c^2 is close to zero so that $A(x, ct)$ is close to $(x + vt, t)$.

Lorentz contraction. If we move a ruler $[a, b]$, then its end coordinates in the cordinate system at rest are not $[a + tv, b + tv]$ as Newtionian mechanics would tell but $[(a + tv)/\gamma, (b + tv)/\gamma]$. This means that the ruler is by a factor $1/\gamma$ larger, when at rest. The constant γ is called the **Lorentz contraction**.
For example, for v equal 2/3 of the speed of light, the contraction is 75 percent. The following picture shows to the left the ruler in the moving coordinate system and to the right in the fixed coordinate system. In the fixed coordinate system, the two end points of the ruler have a different time. (If on the ruler a light signal would be sent out at the same time at the top and at the end, then this signal would reach the origin at different times. The one to the left earlier than the one to the right. Thats why the left point has a smaller time.)





Magnetic force from Electric force.
One striking application of the Lorentz transformation is that if you take two wires and let an electric current flow in the same direction, then the distance between the electrons shrinks: the positively charged ions in the wire see a larger electron density than the ion density. The two wires appear negatively "charged" and repel each other. If the currents flow in different directions and we go into a coordinate system, where the electrons are at rest in the first wire, then the ion density of the ions in the same wire appears denser as well as the electron density in the other wire. The two wires attract each other. The force is proportional to $1/r$.