Lecture 22

Path Independence

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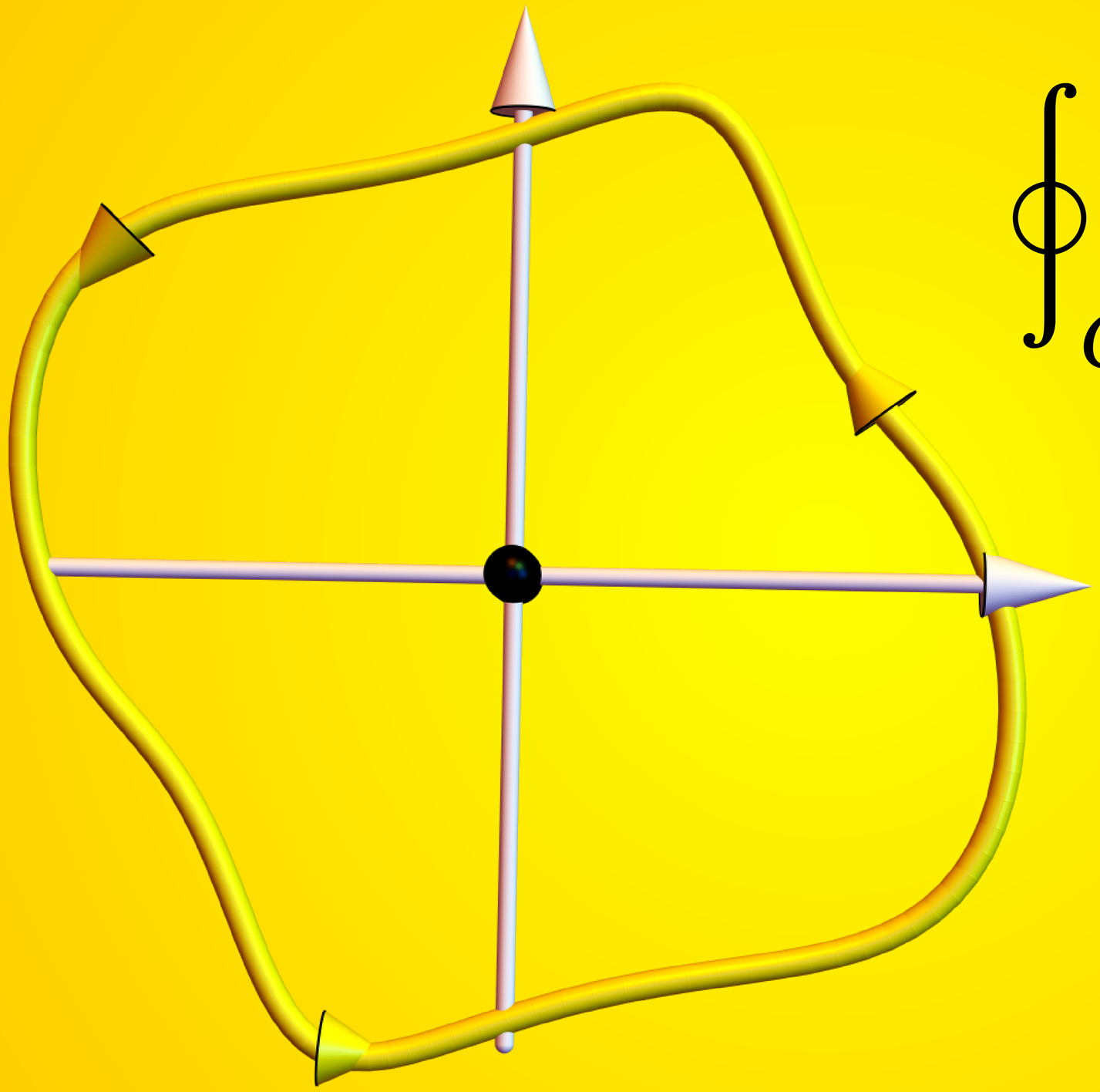
Theorem

1),2),3),4) are all equivalent

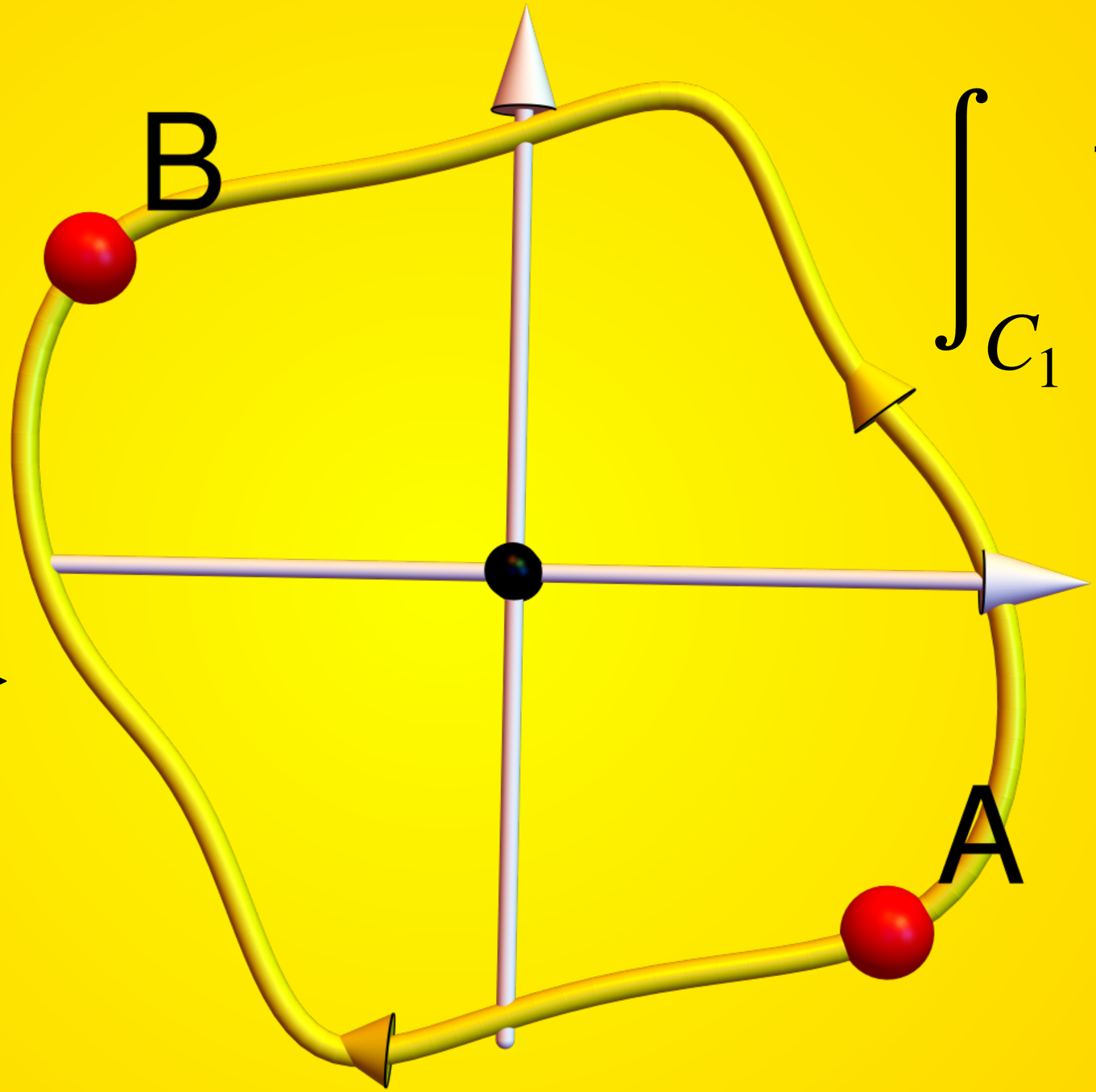
Conservative



$$\oint_C \vec{F} \cdot \overrightarrow{dr} = 0$$

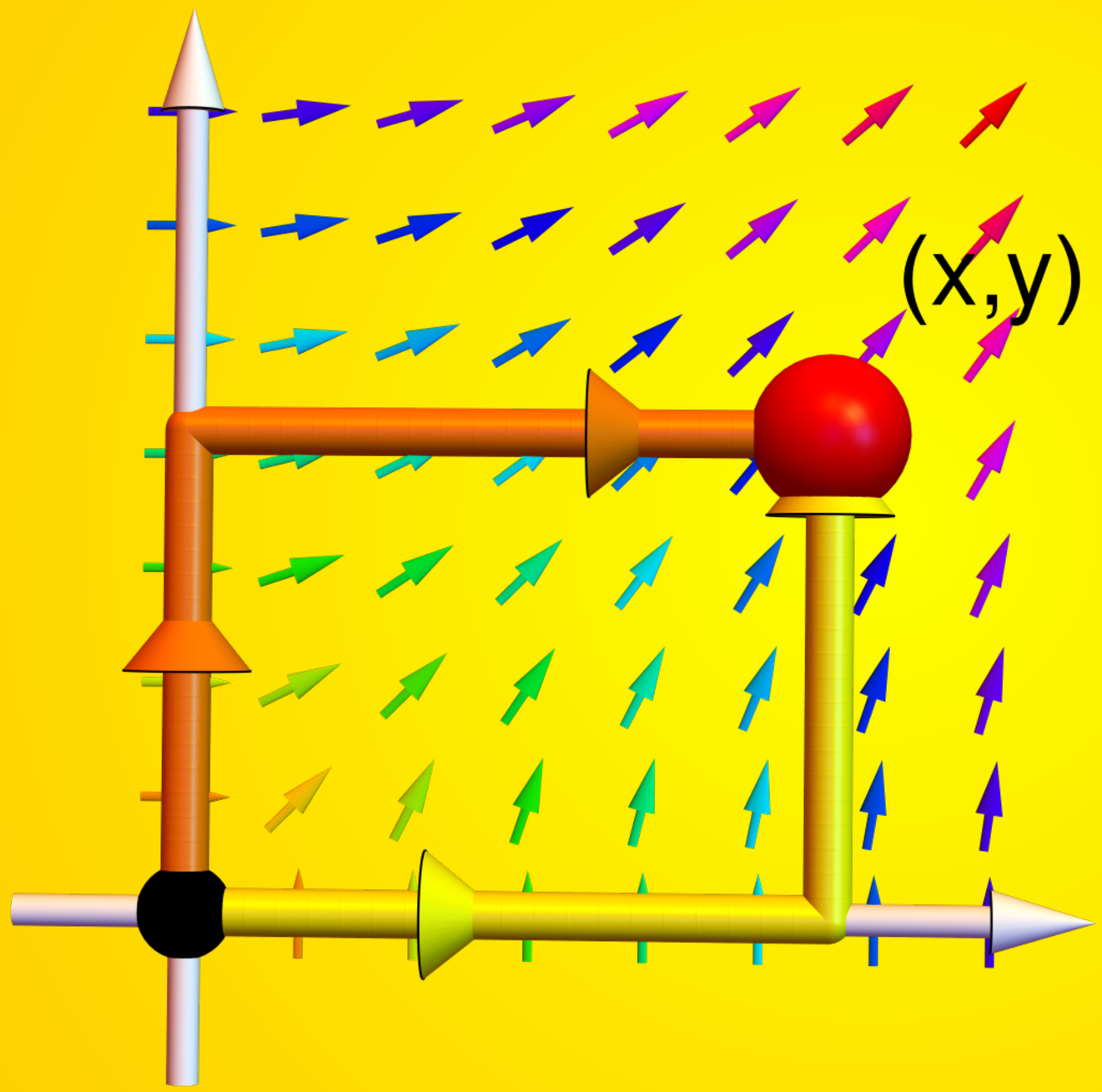
Path independence

$$\int_{C_1} \overrightarrow{F} \cdot \overrightarrow{dr}$$



$$\int_{C_2} \overrightarrow{F} \cdot \overrightarrow{dr}$$

Gradient field

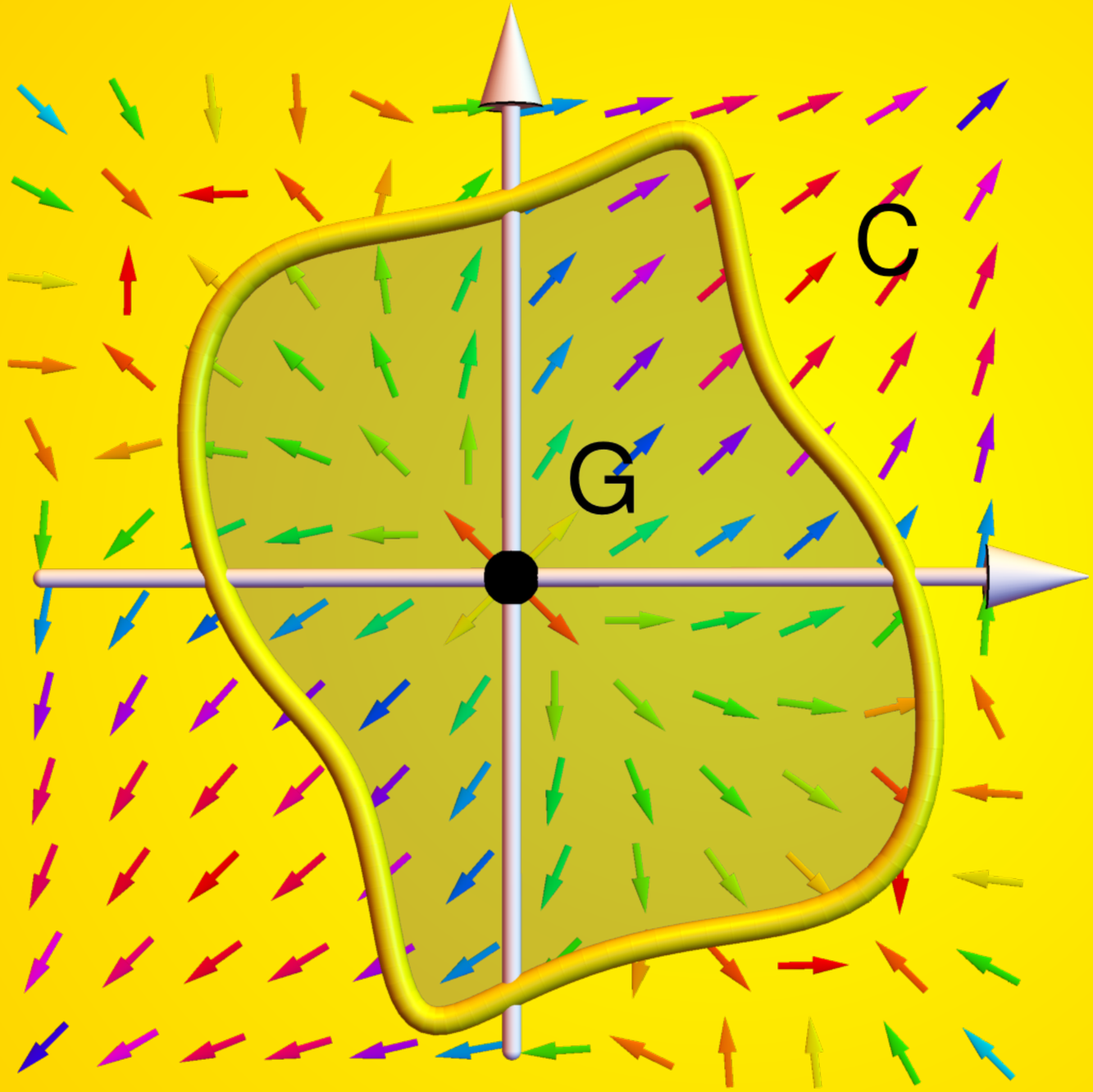


$$\int_{C(x,y)} \overrightarrow{F} \cdot \overrightarrow{dr}$$

$$\overrightarrow{F} = \langle P, Q \rangle$$

$$= \langle f_x, f_y \rangle$$

Irrotational



$$\operatorname{curl}(\vec{F}) = 0$$

everywhere

Theorem

$\vec{F} = \langle P, Q \rangle$

has continuous
partial derivatives

conservative

irrotational

Green

Close

Clairaut

Construct

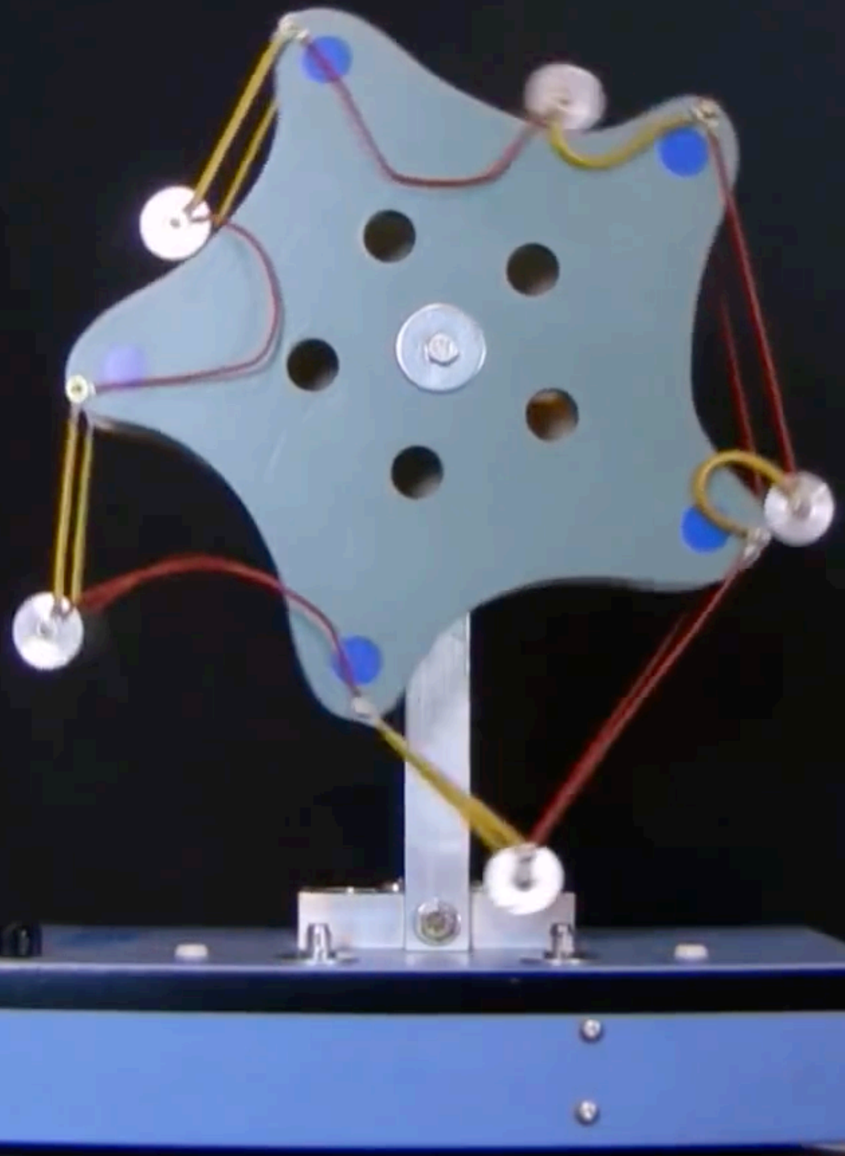
Path independent

gradient field

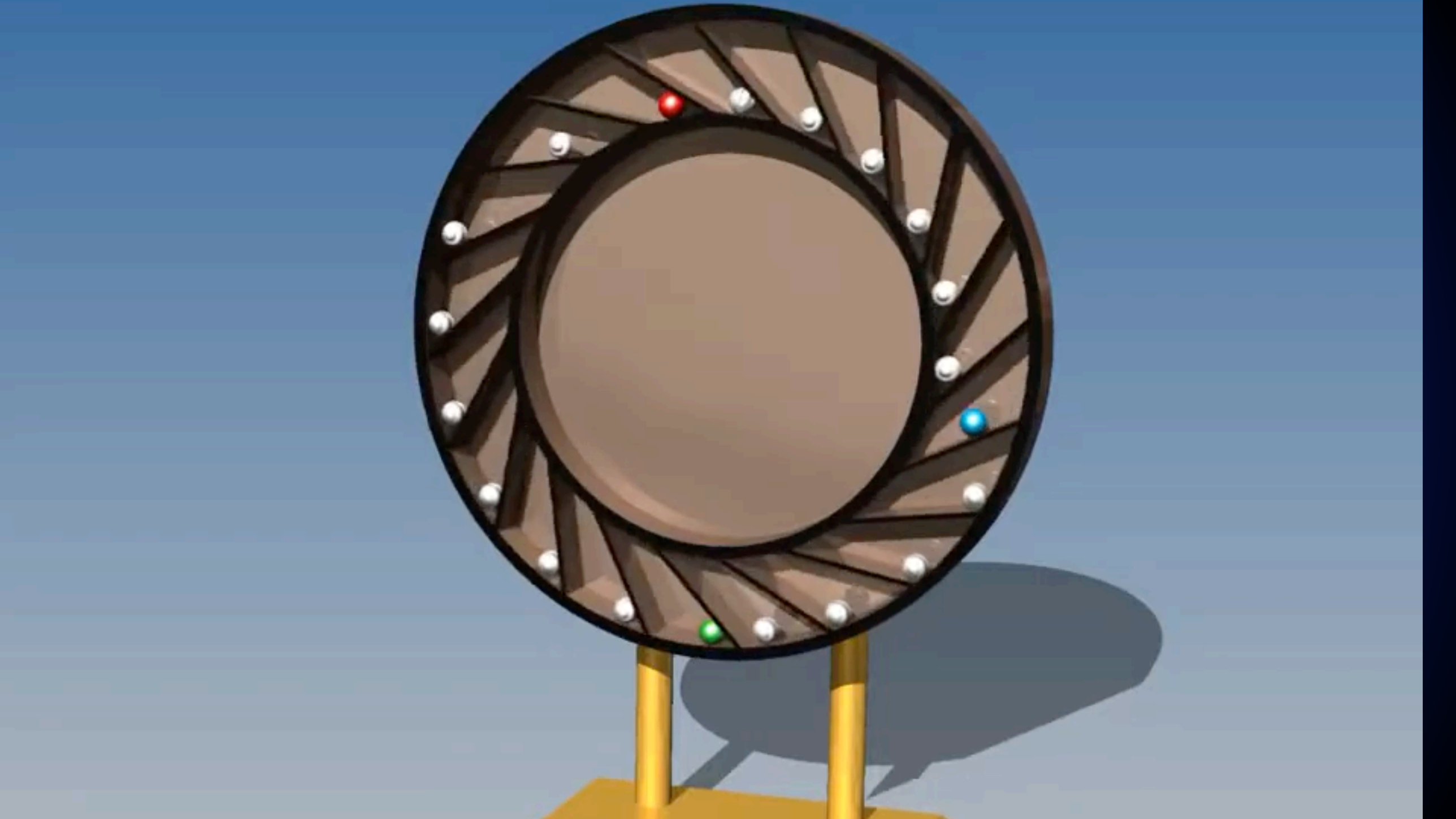
Perpetual motion

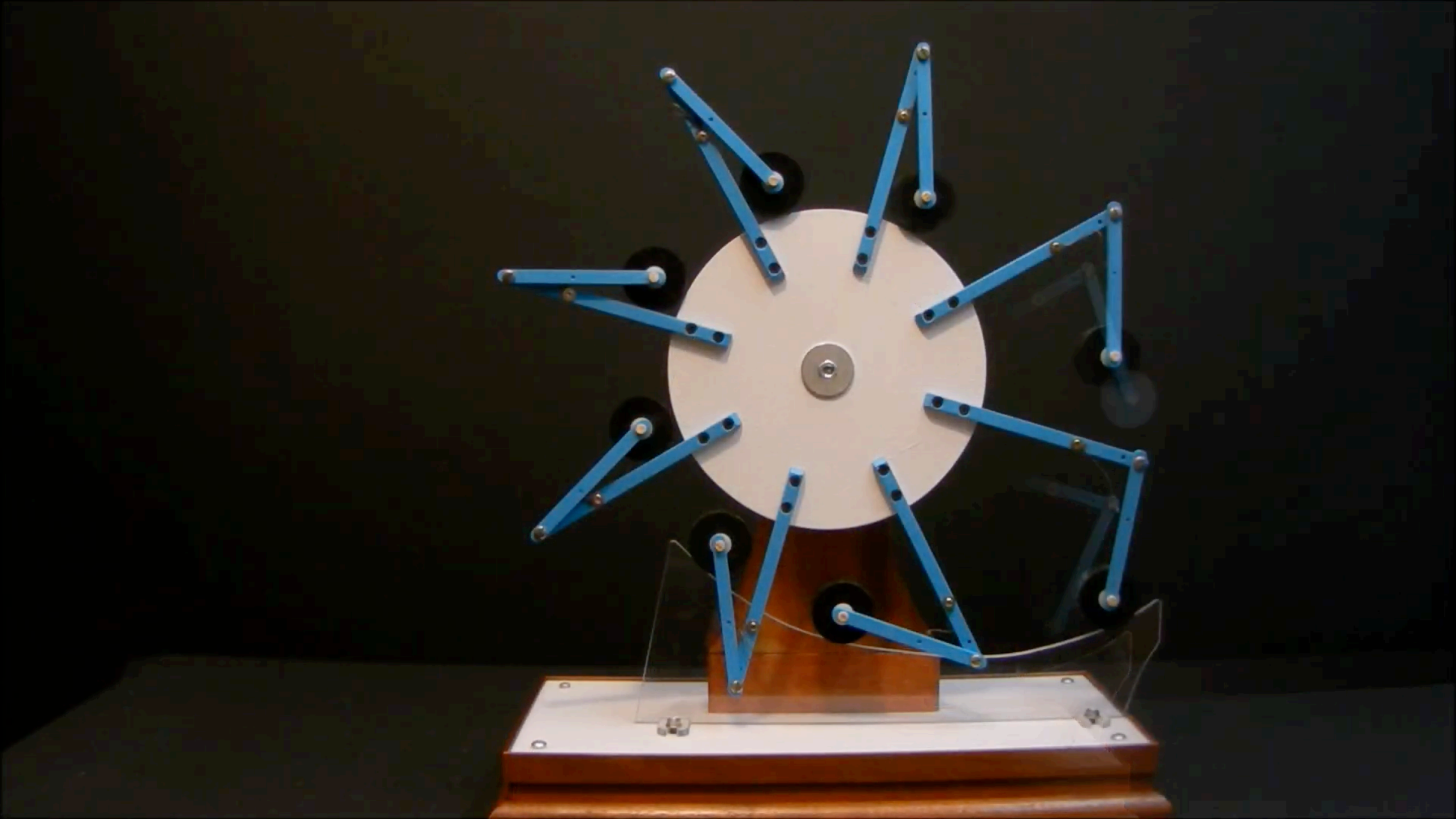
Can we realize a non-conservative vector field?

Leonardo DaVinci

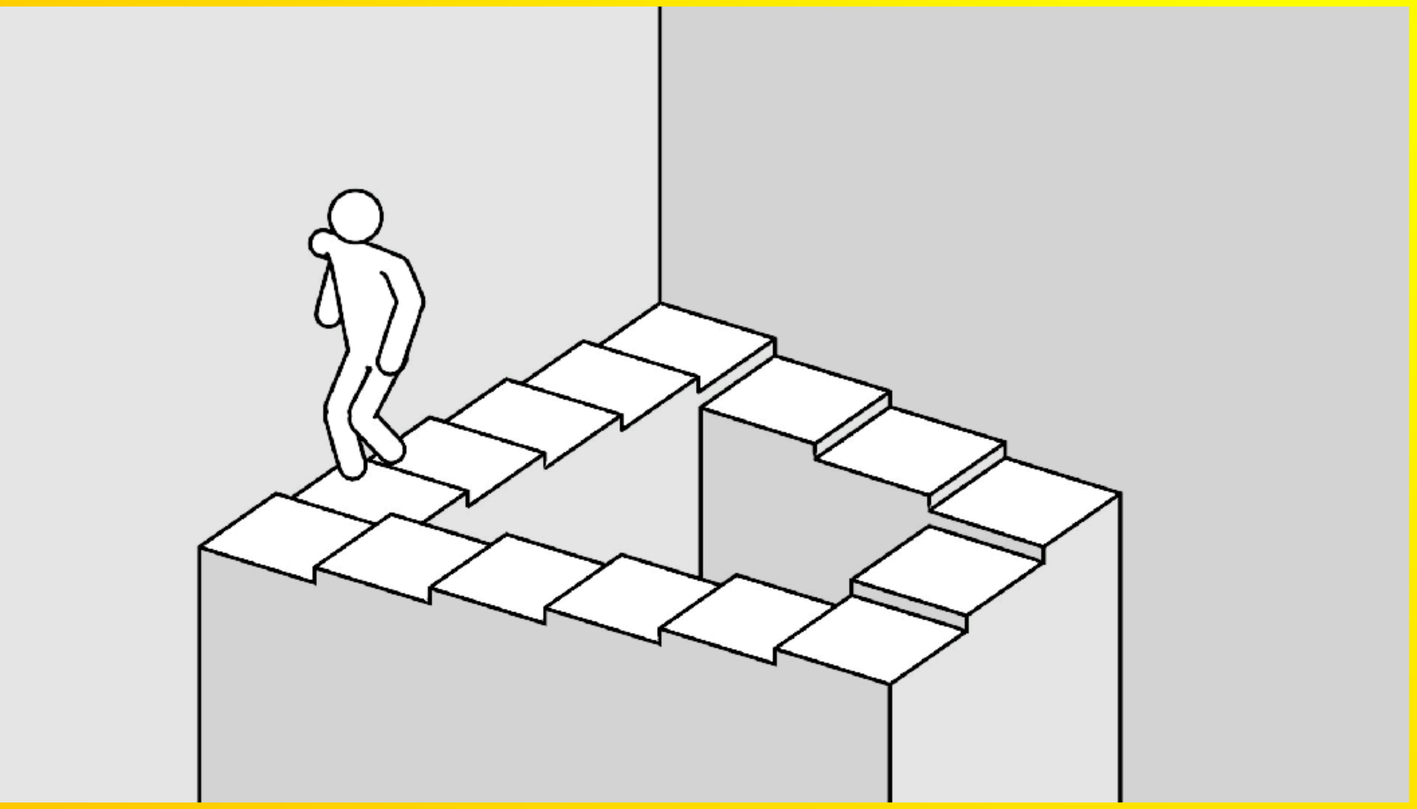


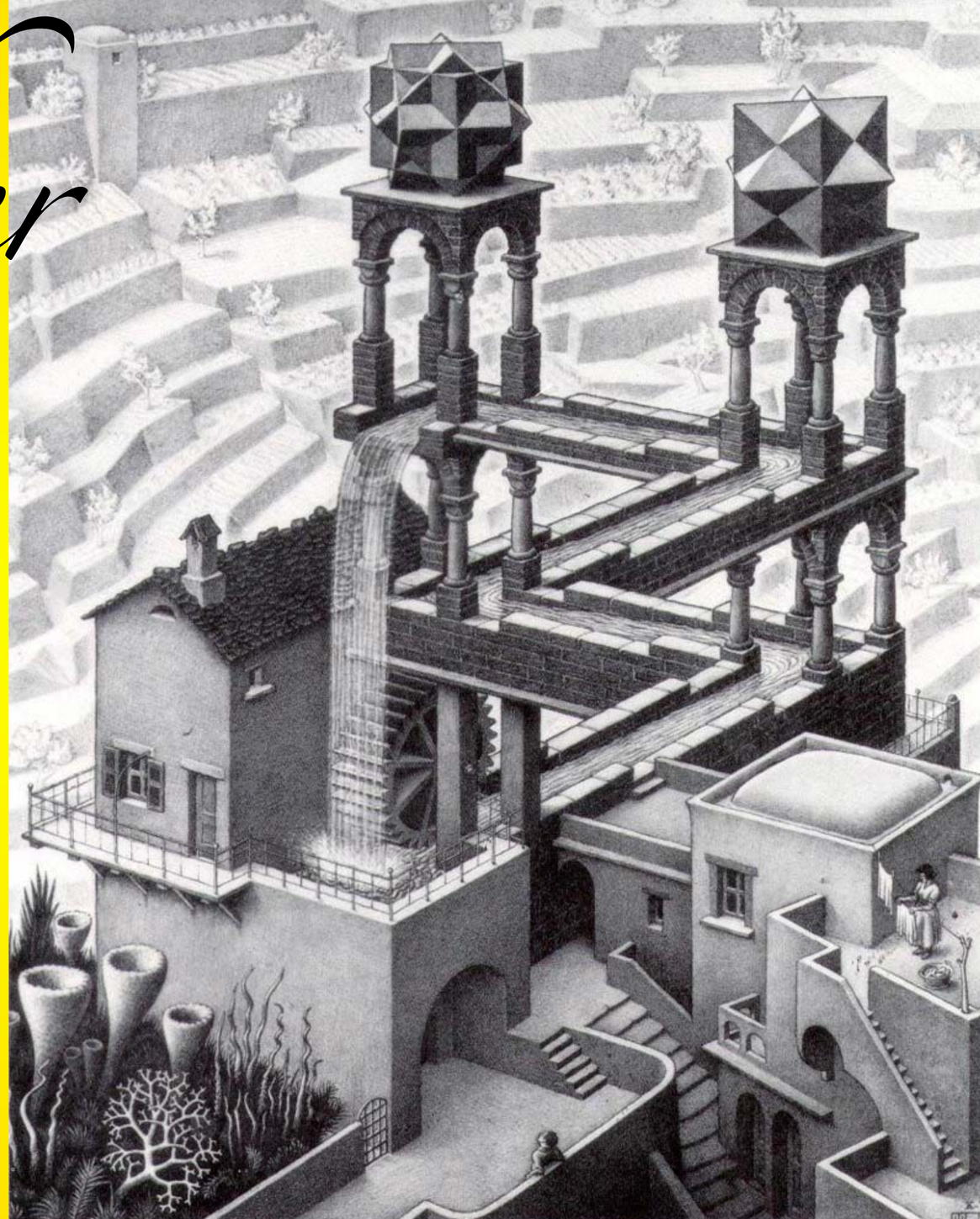

Da Vinci's Wheel

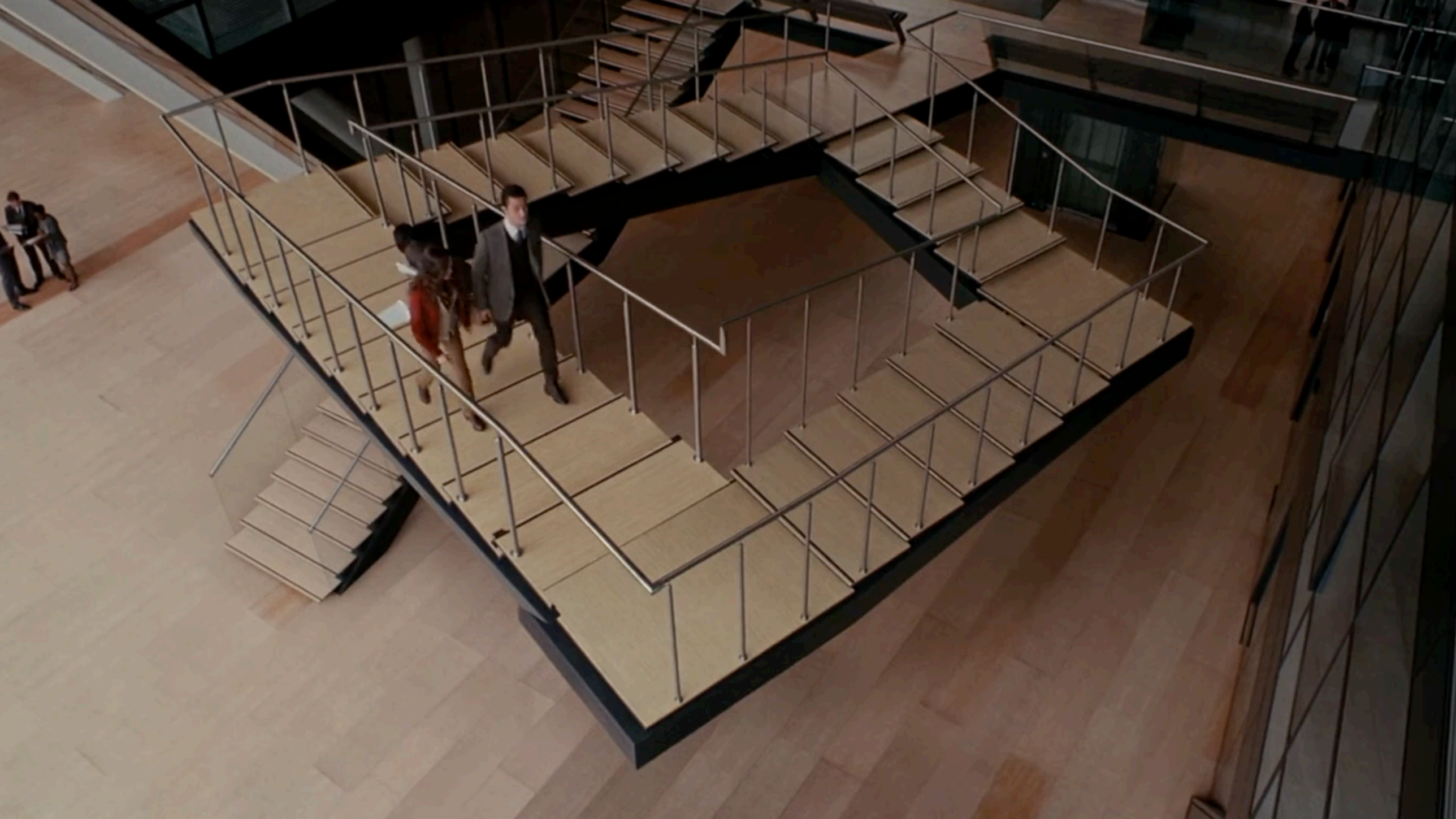


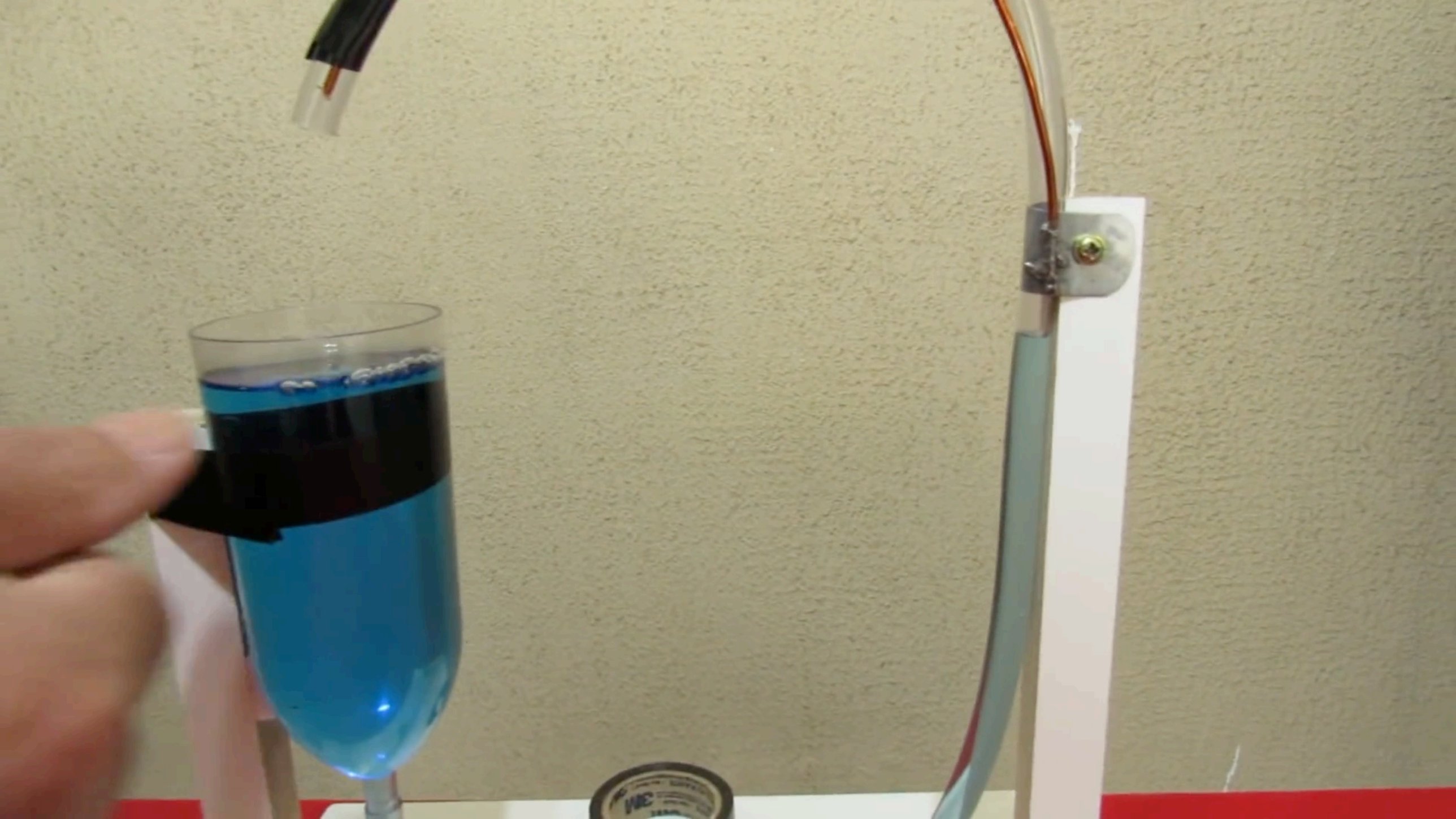


Escher

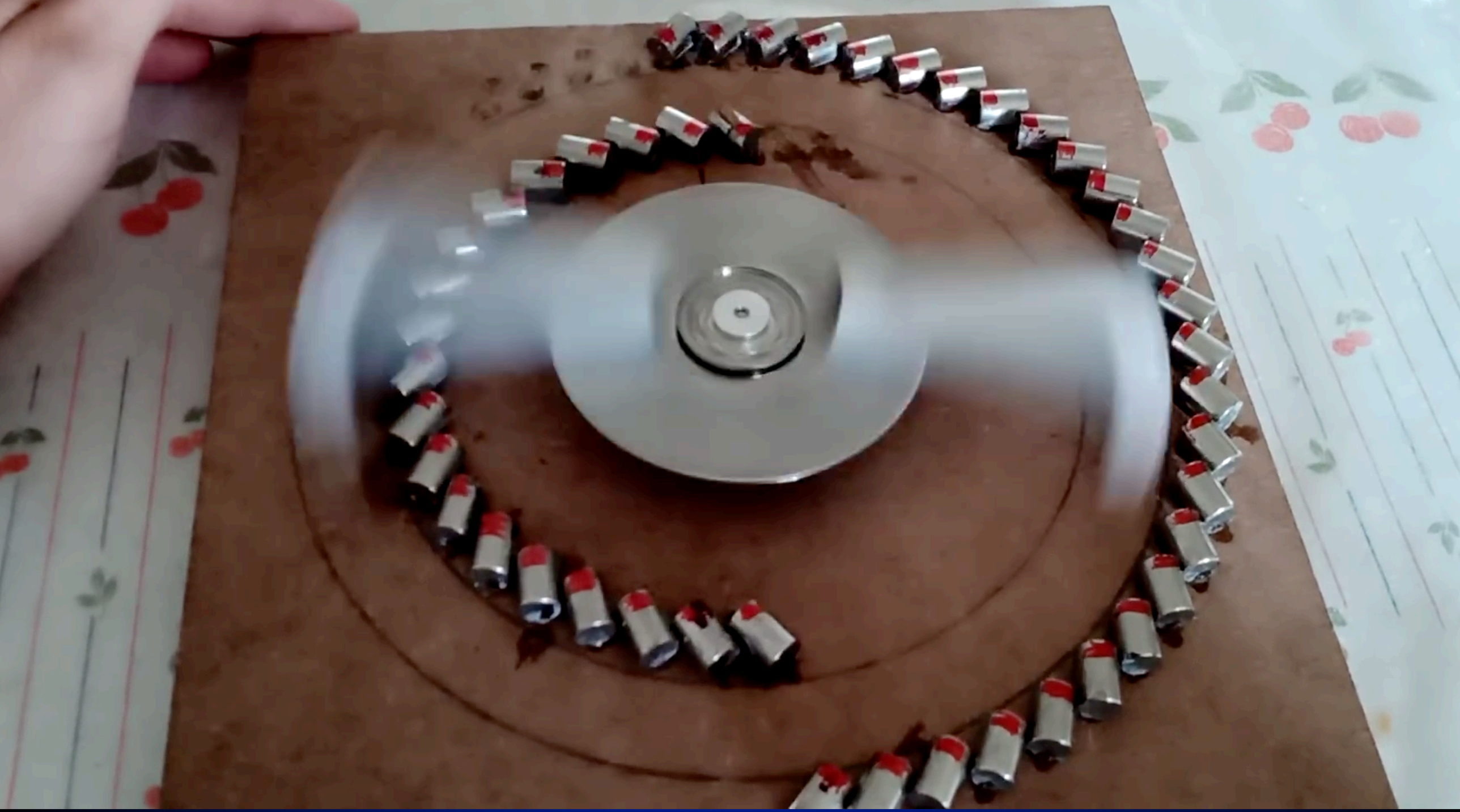


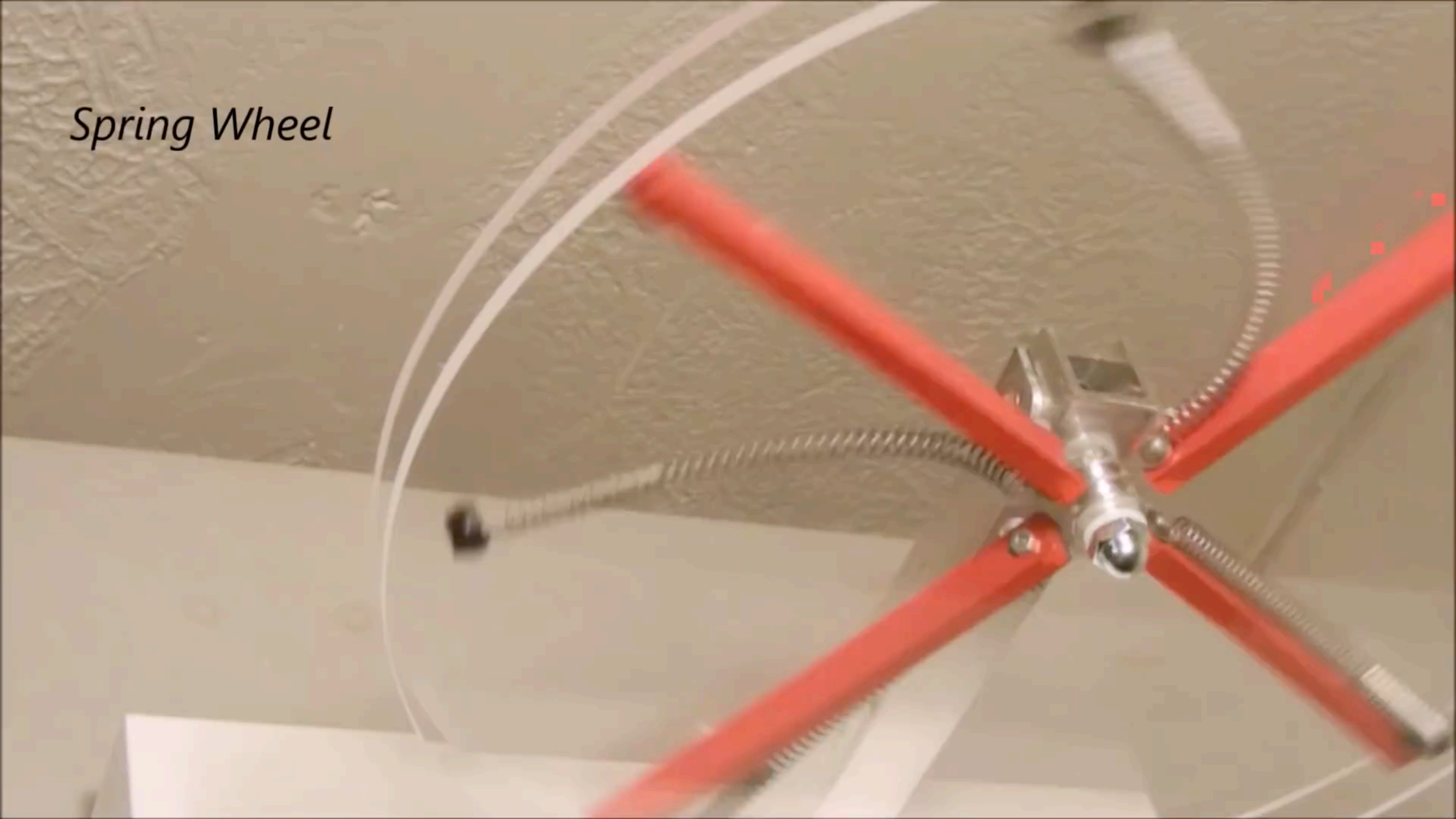







GoPro
TYLER BELL
PME LEGEND
FESTOOL




Spring Wheel

THE END