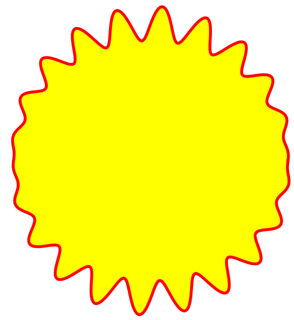
Homework 21: Polar integration

This homework is due Friday, 11/1/2019.

1 Find the area of the **pancake region**



$$\iint_R 1 \, dA \,,$$

where R is given in polar coordinates as $0 \leq r \leq 8 - \sin(\theta)\cos(21\theta)$.

2 Evaluate the following integral by changing to polar coordinates:

$$\iint_R 17x \, dA \,,$$

where R is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$.

3 Use polar coordinates to find the volume of the solid bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

4 a) A city near the sea is modeled by a half disk $D = \{(x, y) \mid x^2 + y^2 \leq 49, x \geq 0\}$. with center the origin and radius 7. What is the average distance of a point in D to the origin? In other words, what is the integral

$$\frac{\iint_D \sqrt{x^2 + y^2} \, dxdy}{\iint_D 1 \, dxdy} \,.$$

b) The distance to the beach is x. Find the average distance

$$\iint_D x \, dxdy / \iint_D 1 \, dxdy$$

to the beach.

5 Evaluate the iterated integral

$$\int_0^2 \int_0^{\sqrt{2x - x^2}} 9\sqrt{x^2 + y^2} \, dy \, dx \,.$$

Main definitions

Polar coordinates $(x, y) = (r\cos(t), r\sin(t))$ allow to describe regions bound by polar curves $(r(\theta), \theta)$.

The **average** of a quantity $f(x, y)$ over a region G is the fraction

$$\frac{\iint_G f(x, y)\ dA}{\iint_G 1\ dA} .$$

To integrate in polar coordinates, we evaluate the integral

$$\iint_R f(x, y)\ dxdy = \iint_R f(r\cos(\theta), r\sin(\theta))r\ drd\theta ,$$

where R is described in polar coordinates.

Example:

To integrate $f(x, y) = x^2 + y^2$ over the region $x^2 + y^2 \leq 9, x \geq 0, y \geq 0$, we integrate

$$\int_0^{\pi/2} \int_0^3 r^2 \cdot r\ drd\theta = (\pi/2)(3^4/4) = 81\pi/8 .$$