Homework 19: Global extrema

This homework is due Monday, 10/28/2019.

1 a) We suppose that the Cobb Douglas production formula $Q(L,K) = L^{1/4}K^{3/4} = 100$, which tells that the quantity Q is constant. What values of L and K minimizes the cost function $C(L,K) = 4L + 5K$ under the constraint $Q(L,K) = 100$?
b) Is there a global maximum or minimum for $C(L,K)$ on the region $L \geq 0, K \geq 0$ without the constraint $Q = 100$? If yes, what is the maximum, or what is the minimum?

2 The extremal value theorem assures that on $D = \{x^2+y^2 \leq 1\}$, a continuous function (and especially a differentiable function) has both a at least one maximum and at least one minimum on D. On an open disk essentially anything goes, as you can see here:
a) Find a differentiable $f(x,y)$ which has exactly one maximum and no minimum on $x^2 + y^2 < 1$. b) Engineer a differentiable function $f(x,y)$ which has exactly one maximum and exactly one minimum on $x^2+y^2 < 1$ and no saddle point and no other critical point. c) Engineer a differentiable function $f(x,y)$ which has exactly two maxima and no other critical point on $x^2+y^2 < 1$.

3 Find the absolute maximum and minimum values of

$$f(x,y) = e^{-x^2-y^2}(x^2 + 2y^2) \quad ,$$

on the disk $D = \{x^2 + y^2 \leq 4\}$.

4 a) Use Lagrange multipliers to find the maximum and minimum values of f subject to the given constraint

$$f(x,y) = \frac{1}{x} + \frac{1}{y};\; g(x,y) = \frac{1}{x^2} + \frac{1}{y^2} = 1 \; .$$

b) Is there a global maximum or global minimum of f on $g = 1$? Is this a case for the Bolzano theorem?

5 A package in the shape of a rectangular box can be mailed by the **US Postal Service** if the sum of its length and girth (the perimeter of a cross-section perpendicular to the length) is at most 108. Find the dimensions of the package with largest volume $V(x, y, z) = xyz$ that can be mailed under the constraint $x + 2y + 2z \leq 108$.

Main definitions:

Standard assumption is still that all functions have continuous first and second derivatives. Maximum means local maximum etc. A point (x_0, y_0) is an **absolute maximum = global maximum** on a domain R, if $f(x, y) \leq f(x_0, y_0)$ for all (x, y) in R.

To find a global maximum, we look at the local maxima and minima as well as the maxima and minima on the boundary. The later is a Lagrange problem. If the domain is unbounded, we also have to look at the behavior of the function when $x, y \to \infty$.

Extremal value theorem of Bolzano: On a bounded closed region or bounded closed curve, there is always a global maximum and global minimum.