Homework 17: Extrema

This homework is due Wednesday, 10/23

1 Find the local maximum and minimum values of the function

$$f(x,y) = 8 + 4xy^2 + \frac{16}{x} + \frac{4}{y}.$$

Use the second derivative test to justify your answer.

2 Classify the critical points of the function

$$f(x,y) = 9e^{2y}(4y^2 - x^2)$$

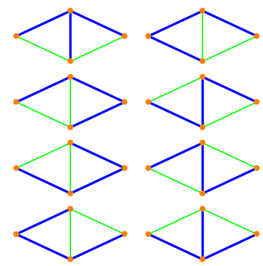
using the second derivative test.

3 Find the local maximum and minimum values and saddle point(s) of the function.

$$f(x,y) = -12\sin x \sin y, -\pi < x < \pi, -\pi < y < \pi$$

4 Companies like **Netflix** or **Hulu** track movie preferences. One can visualize preferences on parameter spaces which is the **intelligence - emotion** plane. Based on viewing habits, the service decides what you want to see. Your profile is a function $f(x,y)$. Maximizing this function allows the company to pick movies for you. a) Assume that your user profile is the function $f(x,y) = -2x^3 + 9x^2 - 12x - y^2$. Find and classify all the critical points and especially find the local maxima of f.
b) Use a computer algebra system to find how many complex critical points the function $f(x,y) = 4x + 3y + x^3 + y^3 - x^4y - x^2y^2 + xy$ has. Locate the real ones and tell whether they are maxima, minima or saddle points.

Graph theorists look at the **Tutte polynomial** $f(x,y)$ of a network. We work with the Tutte polynomial



5 $$f(x,y) = x + 2x^2 + x^3 + y + 2xy + y^2$$

of the **Kite network**. Classify the two critical points using the second derivative test.

Remark. The polynomial is useful: $xf(1-x,0)$ tells in how many ways one can color the nodes of the network with x colors and $f(1,1)$ tells how many spanning trees there are. This picture illustrates that the number of spanning trees of the kite graph is $f(1,1) = 8$ as you see the 8 possible trees.

Main definitions

Standard assumption is that functions are smooth in the sense that all first and second partial derivatives are continuous.

A point (x_0, y_0) is a **critical point** of f if $\nabla f(x_0, y_0) = [0, 0]$.

Fermat's theorem: if f has a local maximum or local minimum at (x_0, y_0) then (x_0, y_0) is a critical point

Second derivative test: Assume (x_0, y_0) is a critical point. Define the discriminant $D = f_{xx}f_{yy} - f_{xy}^2$. If $D < 0$ then it is a saddle point. If $D > 0$, $f_{xx} < 0$ then (x_0, y_0) is a local maximum. If $D > 0$, $f_{xx} > 0$ then (x_0, y_0) is a local minimum.