Homework 1: Geometry and Distance

This homework is due on Wednesday, 9/11 at the beginning of class.

1 a) Find its center and radius of the sphere S given by $x^2 - 4x + y^2 + 2y + z^2 - 8z = 30$.
b) Find the distance from the center of S defined in a) to the sphere $T : x^2 + y^2 + z^2 = 900$.
c) Find the minimal distance between the spheres S and T. This is the minimal distance between two points x, y where x is in S and y is in T.

2 a) Find the distance from $P = (-21, -7, -6)$ to $y = 0$.
b) Find the distance from P to the x-axes.
c) Find a point which has distance 5 from the x axes and distance 2 from the yz-plane.

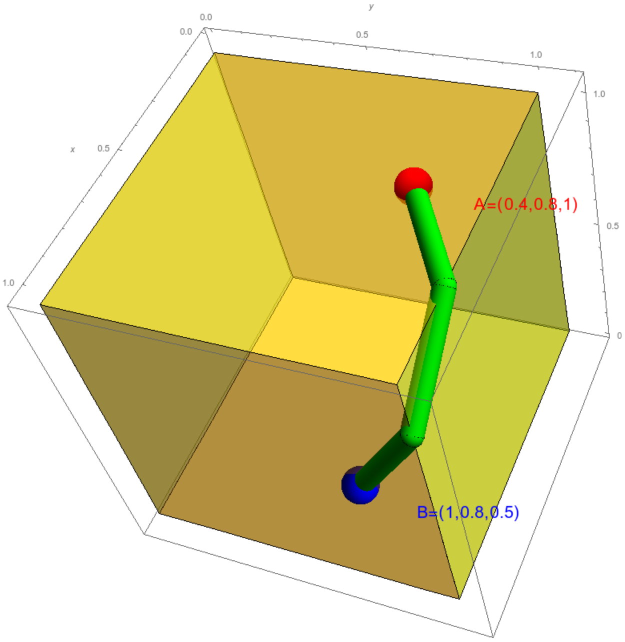
3 a) Find an equation of the largest sphere with center $(4, 11, 9)$ that is contained in the first octant $\{x \geq 0, y \geq 0, z \geq 0\}$.
b) Find the equation for the sphere centered at $(6, 10, 8)$ which passes through the center $(4, 11, 9)$ of the sphere in a).

4 a) Describe the surface S given by $(x - 2y + z)^2 = 4$ in $\mathbb{R}^3$.

If you like to see a bit of a story behind this, on the website, under "data", you find something about prime numbers related to this surface S.

b) What surface is $x^2 + y^2 - 3 = 0$ in $\mathbb{R}^3$?
c) What is the set $x^2 + y^2 - 3 = 0$ in $\mathbb{R}^2$?

5 An ant moves on the unit cube bound by the walls $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ from the point $A = (0.4, 0.8, 1)$ to the point $B = (1, 0.8, 0.5)$. Compute the length of the two obvious paths, where one passes over three faces, the other only over two. Which one is shorter? See the figures on the third page.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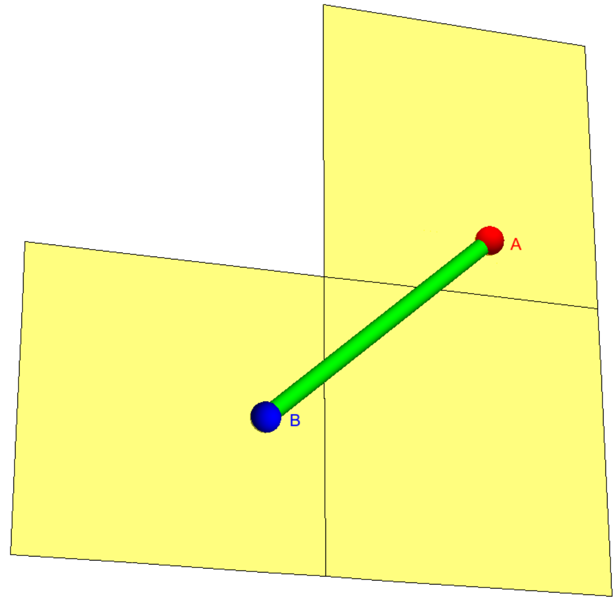
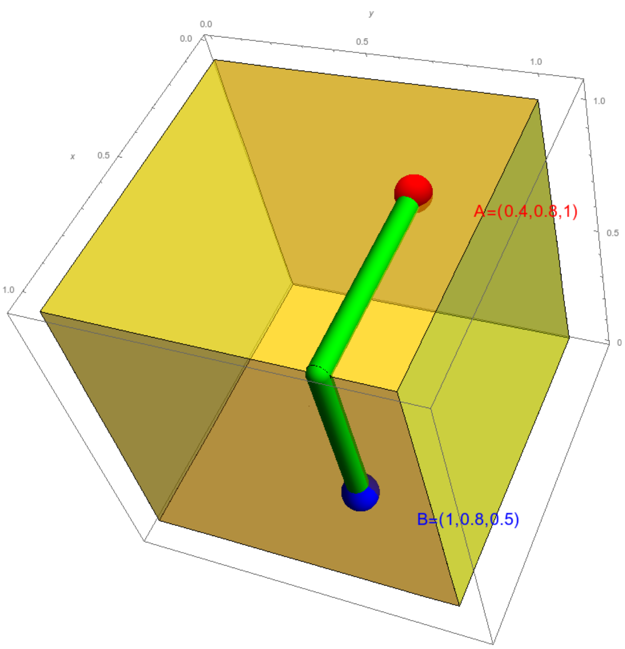
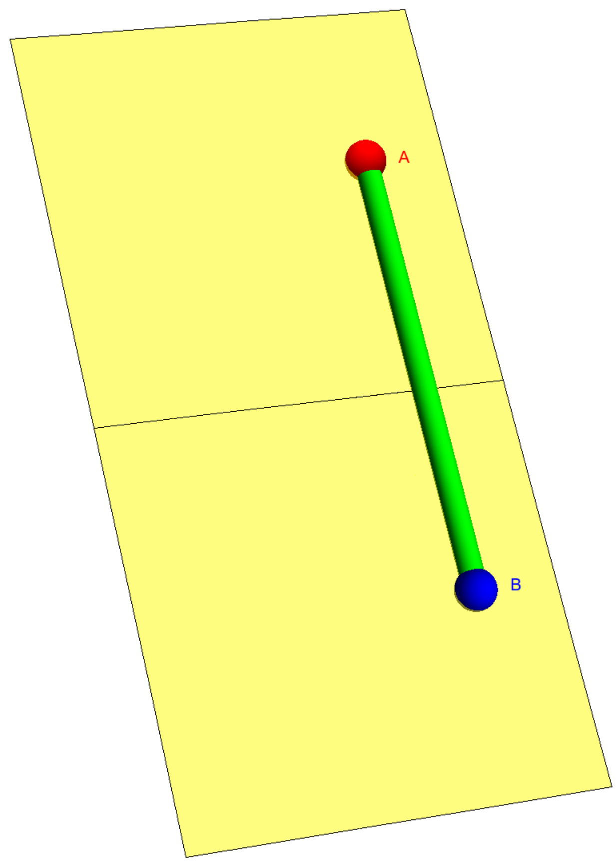
B=(1,0.8,0.5)


A
B


A=(0.4,0.8,1)
B=(1,0.8,0.5)


A
B

Main definitions

Points in the **plane** $\mathbb{R}^2$ or **space** $\mathbb{R}^3$ are described using **coordinates** $P = (x, y)$ or $P = (x, y, z)$. Their signs define **quadrants** in the plane or **octants** in space, regions which intersect at the **origin** $O = (0, 0)$ or $O = (0, 0, 0)$ and are separated by **coordinate planes** $\{x = 0\}, \{y = 0\}, \{z = 0\}$ intersecting in **coordinate axes** like the z-axes $\{y = 0, x = 0\}$.

The **Euclidean distance** between two points $P = (x, y, z)$ and $Q = (a, b, c)$ in space is defined as $d(P, Q) = \sqrt{(x - a)^2 + (y - b)^2 + (z - c)^2}$. The distance between a point P and a geometric object S is the minimal distance $d(P, Q)$ with Q located on S.

A **circle** of radius r centered at $P = (a, b)$ is the set of points in the plane which have distance r from P. A **sphere** of radius ρ centered at $P = (a, b, c)$ is the set of points in space which have distance ρ from P. The equation of a sphere is $(x - a)^2 + (y - b)^2 + (z - c)^2 = \rho^2$.

To **complete the square** of $x^2 + bx + c = 0$, add $(b/2)^2 - c$ on both sides to get $(x + b/2)^2 = (b/2)^2 - c$. Solving for x gives $x = -b/2 \pm \sqrt{(b/2)^2 - c}$. **Example:** $x^2 + 8x + y^2 = 9$. **Solution:** Add 16 on both sides to get $x^2 + 8x + 16 + y^2 = 25$ which is $(x + 4)^2 + y^2 = 25$, a circle of radius $r = 5$ centered at $(-4, 0)$.