

Lecture 32: Overview

1 We consider the following objects:

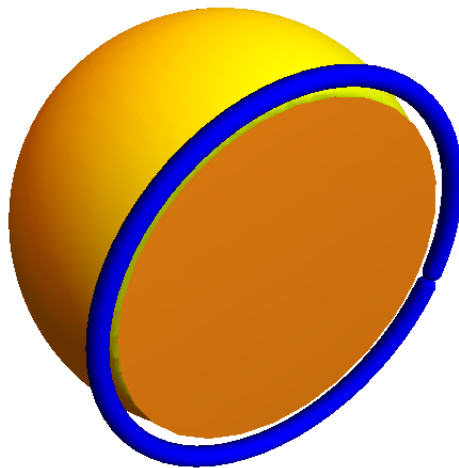
Assume $\vec{F} = [P, Q, R]$ is a vector field in space.

Let E be the solid upper half ball $z \geq 0, x^2 + y^2 + z^2 \leq 1$.

Let S be the upwards oriented half sphere $z > 0, x^2 + y^2 + z^2 = 1$

Let D be the upwards oriented disc $z = 0, x^2 + y^2 \leq 1$

Let C be the curve $\vec{r}(t) = [\cos(t), \sin(t), 0]$



2 Define the following integrals:

$$\text{I)} \quad \int \int \int_E \operatorname{div}(\vec{F}) \, dV$$

$$\text{II)} \quad \int \int_S \vec{F} \cdot d\vec{S}$$

$$\text{III)} \quad \int_C \vec{F} \cdot \boxed{\text{dr}}$$

$$\text{IV)} \quad \int \int_D \vec{F} \cdot d\vec{S}$$

$$\text{V)} \quad \int \int_S \operatorname{curl}(\vec{F}) \cdot d\vec{S}$$

$$\text{VI)} \quad \int \int_D \operatorname{curl}(\vec{F}) \cdot d\vec{S}$$

3 Complete the following identities. You just have to fill in the signs + or - and give the name of the integral theorem, which justifies the identity:

$$\text{a)} \quad \text{I} = \boxed{} \text{II} + \boxed{} \text{IV} \quad \text{by the } \boxed{\phantom{\text{Gauss}}} \text{ theorem}$$

$$\text{b)} \quad \text{III} = \boxed{} \text{V} \quad \text{by the } \boxed{\phantom{\text{Stokes}}} \text{ theorem}$$

$$\text{c)} \quad \text{III} = \boxed{} \text{VI} \quad \text{by the } \boxed{\phantom{\text{Stokes}}} \text{ theorem}$$