Lecture 31: Divergence theorem

1 Find the flux of the vector field $\vec{F}(x, y, z) = [2x, 3y, 4z + \sin(e^x)]$ through the quarter of the unit sphere in the first quadrant, assuming that the surface is oriented outwards?

2 What is the flux of the vector field $\vec{F}(x, y, z) = [x + y + z, y^2, z + \sin(e^x)]$ through the cylinder $x^2 + y^2 = 1, -1 \leq z \leq 1$, where the solid is oriented outwards?

3 Verify that the flux of the vector field $[0, 0, z]$ through the boundary surface of a solid calculates its volume if the boundary is oriented away from the solid.

4 Is the flux of the vector field $[x, y, z]$ through the doughnut given in cylindrical coordinates as $(r - 2)^2 + z^2 = 1$ positive or negative? The doughnut is oriented outwards.