Lecture 30: Stokes theorem

1 Let S be the lower half sphere of radius 2 oriented upwards and let $\vec{F}$ be the vector field $\vec{F} = [-y, x, z^2 \sin(x^2)]$. Find the flux of the curl of $\vec{F}$ through S.

2 Let $\vec{F}(x, y, z) = [y + x^5, y^2, z]$. Find the line integral of $\vec{F}$ along the curve $\vec{r}(t) = [\cos(t), \sin(t), 0], 0 \leq t \leq 2\pi$ by computing the flux of the curl of $\vec{F}$ through a suitable surface.

3 What is the flux of the curl of $[x^4 + y, x + y, z^2 + y$ through the boundary of the cube $x^2 \leq 1, y^2 \leq 1, z^2 \leq 1$?

4 We have seen that the line integral of a gradient field along a closed path in space is zero. Can we see this with Stokes too?

5 How does Stokes theorem prove that vector field with $\text{curl}(\vec{F}) = [0, 0, 0]$ everywhere in space has the closed loop property?

6 How does Stokes theorem prove that the flux of the curl of a vector field $\vec{F}$ through the surface $x^4 + y^4 + z^4 = 1$ is always zero?