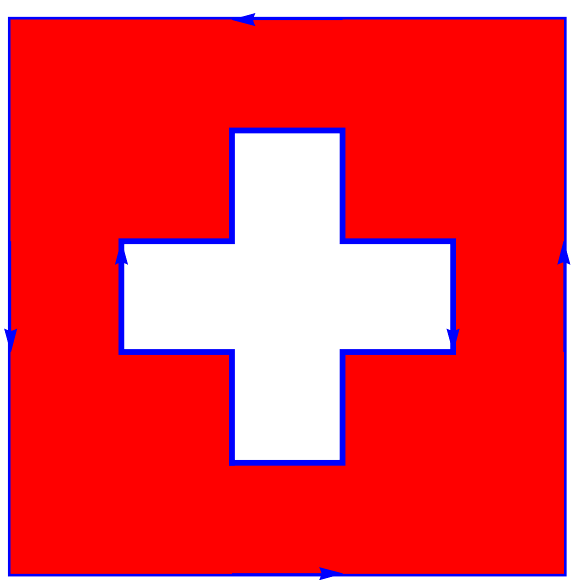
Lecture 28: Greens theorem

1 Let $\vec{F}(x,y) = [x^2 - y, y^2 + x]$ and let $\vec{r}(t)$ be the boundary of the square $0 \leq x \leq 1, 0 \leq y \leq 1$, parametrized counter clockwise.

2 In class we computed the area of the ellipse using $\vec{F}(x,y) = [0, -x]$ and $\vec{r}(t) = [a\cos(t), b\sin(t)]$. It goes faster with $\vec{F}(x,y) = [y, -x]/2$, a field which also has curl constant 1. Do it!

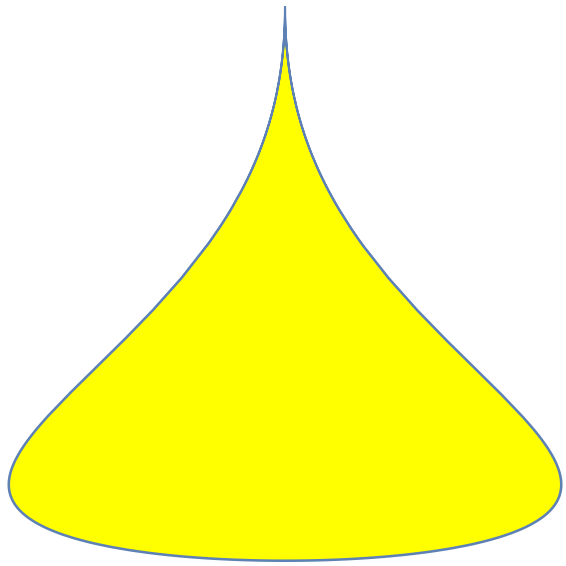
3 Let G be the red complement of the cross in the Swiss flag. The entire flag has dimension 5×5 and the cross consists of 5 squares of unit length. Let C be the boundary of the red complement region oriented so that the region is to the left. The boundary consists of two curves. Find the line integral $\int_C \vec{F}\ d\vec{r}$ for $\vec{F}(x,y) = [x^9 y^{10} + y, y^9 x^{10} - x]$.



4 Find the area of the region enclosed by

$$\vec{r}(t) = [\frac{\sin(\pi t)^2}{t}, t^2 - 1]$$

for $-1 \leq t \leq 1$. Use Greens theorem with $\vec{F} = [0, x]$.



Remarks.

- This problem could not be solved without integral theorem.
- Also $\vec{F} = [-y, 0]$ has curl 1 but the integral would not work.