Lecture 22: Surface area

For a parametric surface, the surface area is defined as

$$\int \int_R |\vec{r}_u \times \vec{r}_v| \cdot dudv \; .$$

For a **surface of revolution** parameterized by

$$\vec{r}(\theta, z) = [g(z)\cos(\theta), g(z)\sin(\theta)] \; .$$

we get

$$|\vec{r}_\theta \times \vec{r}_z| = |g(z)|\sqrt{1 + g'(z)^2} \; .$$

The surface area of such a surface of revolution is

$$2\pi \int_a^b |g(z)|\sqrt{1 + g'(z)^2} \; dz \; .$$

1 Find the surface area of the surface

$$\vec{r}(u, v) = [u, v, 2u]$$

with $0 \leq u \leq 1$ and $0 \leq v \leq 3$.

2 Find the surface area of the surface

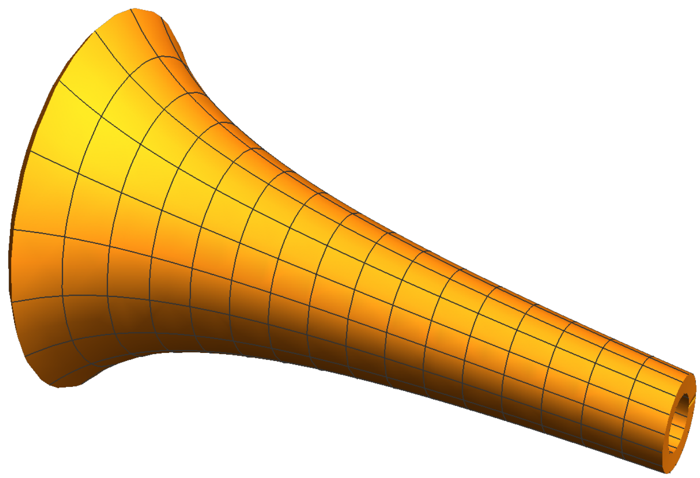
$$\vec{r}(u, v) = [u, v, u^2]$$

where $0 \leq u \leq 1$ and $0 \leq v \leq u$.

Gabriel's trumpet is the surface of revolution where $g(z) = 1/z$, where $1 \leq z < \infty$.

3 Verify that the volume of the trumpet is $\int_1^\infty \pi g(z)^2 \; dz = \pi$.

4 Compute the surface area integral of the trumpet.



We conclude that the trumpet is a surface of finite volume but with infinite surface area! You can fill the trumpet with a finite amount of paint, but this paint does not suffice to cover the surface of the trumpet!