Lecture 21: Polar integration

1 Evaluate the integral

$$\int\int_R \frac{1}{\sqrt{x^2+y^2}}\, dxdy$$

over the disc of radius 1.

2 The integral

$$2\int_0^{\pi/2}\int_\theta^{2\theta} r\, drd\theta$$

computes the area of the region shown below. Can you see why?

3 Find the area.

4 Integrate $f(x,y) = x^2$ over the unit disk $\{2 \leq x^2+y^2 \leq 9\}$.

5 Can you evaluate the following integral? (Halloween problem!)

$$\int_0^1\int_0^{\sqrt{1-\theta^2}} r^2\, drd\theta\ .$$

Hint: Write it in more convenient coordinates:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dydx \; .$$

This is a quarter disc in the x, y plane. Now use polar coordinates.