Lecture 19: Global extrema

The task to find global maxima or global minima can be quite a bit of work. We have to do two things. First find the critical points in the interior, then find the critical points at the boundary using the Lagrange multiplier method.

1 Find the local maxima and minima of the function $f(x, y) = x^2 + y^4 + x - 2y^2$ in the interior of the region $x^2 + y^2 \leq 4$. Classify them using the second derivative test.

This is a bit of review of the Monday lecture on extrema. There will be three points.

Critical point:	$D = f_{xx}f_{yy} - f_{xy}^2$	f_{xx}	nature
$(x, y) =$	$D =$	$f_{xx} =$	
$(x, y) =$	$D =$	$f_{xx} =$	
$(x, y) =$	$D =$	$f_{xx} =$	

2 Find the local maxima and minima of the same function $f(x, y) = x^2 + y^4 + x + 2y^2$ on the boundary of the region $x^2 + y^2 \leq 4$. Hint: there will be 8 points.

3 Compare the list of all critical points, the ones in the interior and the ones on the boundary to find the maximum and minimum.

4 Now forget about the region we have considered initial and look at the function $f(x, y)$ on the entire plane $\mathbb{R}^2$. Does the function $f(x, y)$ have a global maximum on $\mathbb{R}^2$?

5 Does the function $f(x, y)$ have a global minimum on the entire plane $\mathbb{R}^2$?

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[1] $(0,-1),(0,0),(0,1)$ are the critical points and $(0,-2),(0,2),(2,0),(-2,0),(\pm\sqrt{7/2},\pm\sqrt{1/2})$ are the critical points on the boundary.