4/5/2011: Second midterm practice II

Your Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- Except for multiple choice problems, give computations.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) TF questions (20 points) No justifications are needed.

1) T F The formula $\int_0^x f''(x)\,dx = f'(x) - f'(0)$ holds.

2) T F The area of the lower half disc is the integral $\int_{-1}^1 -\sqrt{1-x^2}\,dx$

3) T F If the graph of the function $f(x) = x^2$ is rotated around the interval $[0,1]$ we obtain a solid with volume $\int_0^1 \pi x^4\,dx$.

4) T F The identity $d/dx \int_0^x f''(t)\,dt = f'(x)$ holds.

5) T F There is a point in $[0,1]$, where $f'(x) = 0$ if $f(x) = x^3 - x^2 + 1$.

6) T F The fundamental theorem of calculus assures that $\int_a^b f'(x)\,dx = f(b) - f(a)$.

7) T F If f is differentiable on $[a,b]$, then $\int_a^b f(x)\,dx$ exists.

8) T F The integral $\int_0^{\pi/2} \sin(\sin(x))\,dx$ is positive.

9) T F The anti-derivative of an anti-derivative of f is equal to the derivative of f.

10) T F If a function is positive everywhere, then $\int_a^b f(x)\,dx$ is positive too.

11) T F If a differentiable function is odd, then $\int_{-1}^1 f(x)\,dx = 0$.

12) T F If $f_c(x)$ is a function with a local minimum at 0 for all $c < 0$ and no local minimum in $[-1,1]$ for $c > 0$, then $c = 0$ is called a catastrophe.

13) T F The term "improper integral" is a synonym for "indefinite integral".

14) T F The function $F(x) = x\sin(x)$ is an antiderivative of $\sin(x)$.

15) T F The mean value theorem holds for every continuous function.

16) T F Newton and Leibniz were best buddies all their life. Leibniz even gave once the following famous speech: "You guys might not know this, but I consider myself a bit of a loner. I tend to think of myself as a one-man wolf pack. But when my sister brought Isaac home, I knew he was one of my own. And my wolf pack... it grew by one.

17) T F Any function $f(x)$ satisfying $f(x) > 0$ is a probability density function.

18) T F The moment of inertia integral I can be used to compute energy with the relation $E = \omega^2 I/2$ where ω is the angular velocity.

19) T F If $0 \le f(x) \le g(x)$ then $0 \le \int_0^1 f(x)\,dx \le \int_0^1 g(x)\,dx$.

20) T F The improper integral $\int_0^\infty 1/(x^4+1)\,dx$ is finite.

Problem 2) Matching problem (10 points) No justifications are needed.

From the following functions there are two for which no elementary integral is found. Find them. You can find them by spotting the complement set of functions which you can integrate.

Function	Antiderivative is not elementary	Function	Antiderivative is not elementary
e^{-x^2}		$1/\log(x)$	
$\sin(3x)$		$\tan(3x)$	
$1/x$		$\arctan(3x)$	

Problem 3) Matching problem (10 points) No justifications are needed.

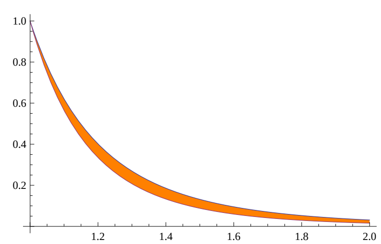
Which of the following problems are related rates problems? Several answers can apply.

Problem	Related rates?
Find the volume of a sphere in relation to the radius.	
Relate the area under a curve with value of the curve.	
If $x^3 + y^3 = 5$ and $x' = 3$ at $x = 1$, find y'.	
Find the rate of change of the function $f(x) = \sin(x)$ at $x = 1$	
Find r' for a sphere of volume V satisfying $d/dtV(r(t)) = 15$.	
Find the inflection points of $f(x) = x^3 + 3x + 4$.	
Find the global maxima of $f(x) = x^4 + x^3 - x$.	

Problem 4) Area computation (10 points)

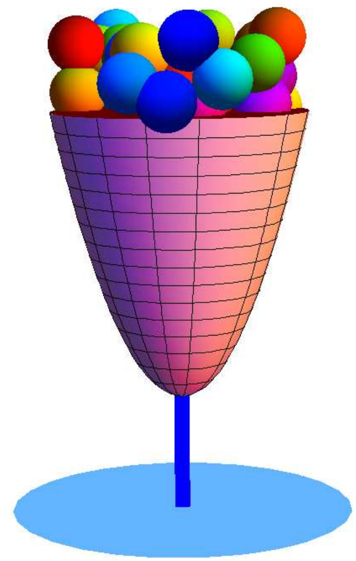
a) (5 points) Find the area of the region enclosed by the curves $3 - x^4$ and $3x^2 - 1$.

b) (5 points) Find the area of the region between $1/x^6$ and $1/x^5$ from $x = 1$ to $x = \infty$.



Problem 5) Volume computation (10 points)

Julian eats some magic "Bertie Botts Every Flavor Beans" from a cup which is a rotationally symmetric solid, for which the radius at position x is $\sqrt{x}$ and $0 \leq x \leq 4$. Find the volume of Julian's candy cup.



Problem 6) Definite integrals (10 points)

Find the following definite integrals

a) (5 points) $\int_1^2 x + \tan(x) + \sin(x) + \cos(x) + \log(x)\ dx$.

b) (5 points) $\int_1^3 (x + 1)^3\ dx$

Problem 7) Anti derivatives (10 points)

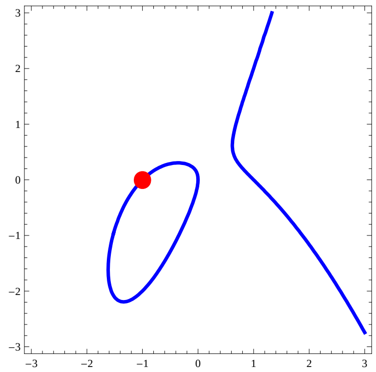
Find the following anti-derivatives

a) (5 points) $\int \sqrt{x^3}\ dx$

b) (5 points) $\int 4/\sqrt{x^5}\ dx$

Problem 8) Implicit differentiation (10 points)

The curve $y^2 = x^3 + 2xy - x$ is an example of an **elliptic curve**. Find dy/dx at the point $(-1, 0)$ without solving for y first.



Problem 9) Applications (10 points)

The probability density of the exponential distribution is given by $f(x) = (1/2)e^{-x/2}$. The probability to wait for for time x (hours) to get an idea for a good calculus exam problem is $\int_0^x f(x)\, dx$. What is the probability to get a good idea if we wait for $T = 10$ (hours)?

Problem 10) Applications (10 points)

What is the **average value** of the function

$$f(x) = 4 + 1/(1 + x^2)$$

on the interval $[-1, 1]$?