MATH 1A - PROBLEM SET 22

* PROBLEM 1

$$\int_0^1 x^{2/3}\,dx$$

$$= \frac{x^{5/3}}{5/3}\Big|_0^1 = \frac{3x^{5/3}}{5}\Big|_0^1 = \frac{3(1)^{5/3}}{5} - \frac{3(0)^{5/3}}{5} = \boxed{\frac{3}{5}}$$

* PROBLEM 2

THE INTEGRAL EXISTS $0 < p < \infty$ WHERE $p \neq 1$

$$\int_1^\infty \frac{1}{x^p}\,dx$$

$$\int_1^\infty x^{-p}\,dx = \left(\frac{x^{-p-1}}{p-1}\right)_1^\infty \Rightarrow \text{THIS CAN BE SOLVED FOR ALL p EXCEPT } \boxed{p=1}$$

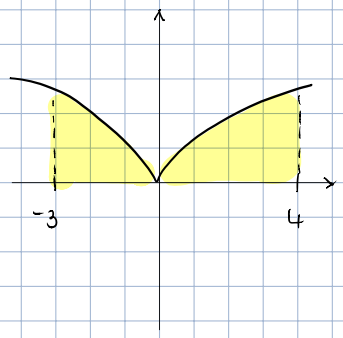
IF $p=1$, $\left(\frac{x^{-1-1}}{1-1}\right) = \frac{x^{-2}}{0}$ (INVALID)

* PROBLEM 3

$$\int_{-1}^1 \frac{1}{\sqrt{1-x^2}}\,dx \Rightarrow \text{THIS IS A STANDARD INTEGRAL}$$

$$\int_{-1}^1 \arcsin(x)\,dx = \arcsin(1) - \arcsin(-1) = \boxed{\pi}$$

* PROBLEM 4



$$\int_{-3}^4 (x^2)^{1/3}\,dx$$

$$\int_{-3}^4 x^{2/3}\,dx$$

$$\frac{x^{5/3}}{5/3}\Big|_{-3}^4 = \frac{3x^{5/3}}{5} = \frac{3(4)^{5/3}}{5} - \frac{3(-3)^{5/3}}{5} = \boxed{9.79}$$

CHECKING $\int_{-3}^0 (x^2)^{1/3}\,dx$ $\qquad$ $\int_0^4 (x^2)^{\frac{1}{3}}\,dx$

$$\frac{3(0)^{5/3}}{5} - \frac{3(-3)^{5/3}}{5} = 3.744 \qquad \frac{3(4)^{\frac{5}{3}}}{5} - \frac{3(0)^{5/3}}{5} = 6.047$$

$3.744 + 6.047 = \boxed{9.79}$

* PROBLEM 5

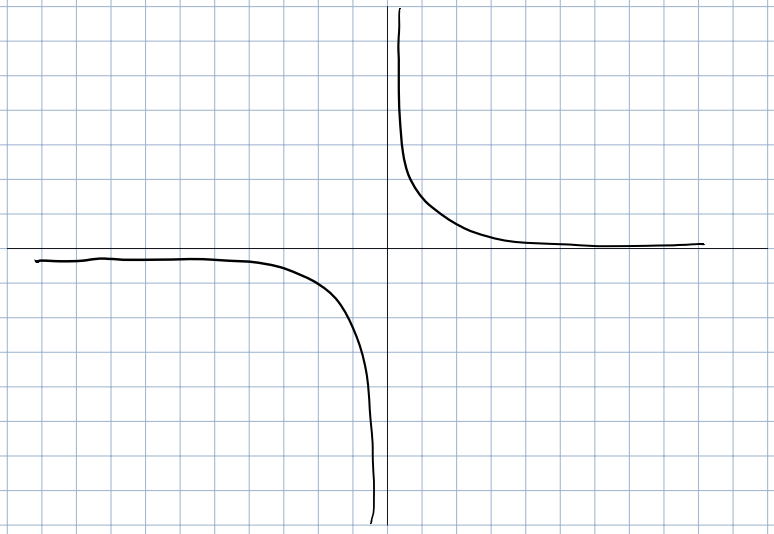
$$\int_{-4}^{5} \frac{3}{x^3}\,dx$$

$$3\int_{-4}^{5} \frac{1}{x^3}\,dx$$

$$3\int_{-4}^{5} x^{-3}\,dx$$

$$3\int_{-4}^{5} \frac{x^{-2}}{-2}\,dx$$

$$3\left(-\frac{1}{2x^2}\right) = \frac{-3}{2x^2}\Bigg|_{-4}^{5}$$



=> SINCE THE INTEGRAL IS DIVERGENT WE WILL TAKE A POSITIVE

$$\frac{-3}{2x^2}\Bigg|_{1}^{5} \qquad \frac{-3}{2x^2}\Bigg|_{-4}^{-1}$$

$$\frac{-3}{2(5^2)} - \frac{(-3)}{2(1^2)} = \boxed{1.44} \qquad \frac{-3}{2(-1)^2} - \frac{(-3)}{2(-4)^2} = \boxed{-1.406}$$

$$\oplus = \boxed{0.034} \Rightarrow \text{CAUCHY PRINCIPAL VALUE}$$