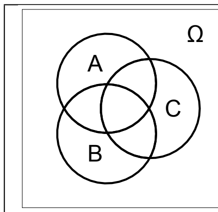
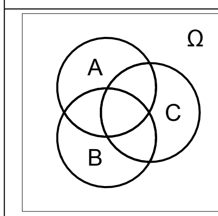
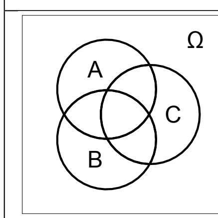
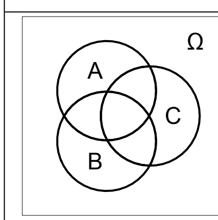
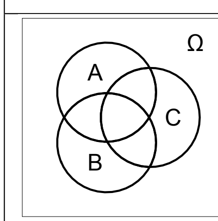
PROBABILITY THEORY

MATH 154

Inclass Midterm

You have 40 minutes. No material, no tools. Each problem is 10 points.

Problem 1) A, B, C are events. Shade the regions in the Venn diagram. Shade the regions. We use the notation from class for the Boolean algebra.

	$(A \setminus B) \cdot C$
	$A^c \cdot B$
	$A \cdot A \cdot A + B + C$
	$(\Omega + A^c) \cap A$
	$(A \setminus B) + (B \setminus A)$

Problem 2) A random variable X takes the value $a_1 = 7$ with probability $p = 1/4$ and the value $a_2 = -7$ with probability $p_2 = 1/4$ and $a_3 = 0$ with probability $p_3 = 1/2$.

a) What is the expectation m of X?

b) What is the standard deviation $\sigma[X]$ of X?

c) What is the skewness $\mathrm{E}[(X - m)^3)]/\sigma^3$?

d) Write down the moment generating function $M_X(t)$.

e) And finally give the characteristic function $\phi_X(t)$.

Problem 3) Let $(\Omega, \mathcal{A}, \mathrm{P})$ be the Lebesgue probability space on $\Omega = [-1, 1]$ with the standard measure $\mathrm{P}[[a, b)] = (b - a)/2$. Let X be the random variable $X(x) = x^3$ and Y the random variable $Y(x) = x$.

a) What are the expectations of X and Y?

b) What are the standard deviations $\sigma[X], \sigma[Y]$ of X and Y?

c) What is the moment generating function $M_Y(t)$?

d) What is the CDF $F_X(t)$?

e) What is the correlation $\mathrm{Corr}[X, Y]$ between X and Y?

Problem 4) True/False Problems.

1) True or False? There is a σ algebra with 16 events.

2) True or False? There is a bijection between the complex numbers and the natural numbers

3) True or False? If you know the moment generating function of a bounded random variable X, then you know all the moments of X.

4) True or False? If A is an event of probability $\mathrm{P}[A] = 0$, then A is the empty set.

5) True or False? On a Lebesgue probability space there are random variables taking only finitely many values.

6) True or False? If every function $\Omega \to \mathbb{R}$ is a random variable, then the σ-algebra is trivial.

7) True or False? If A is an event in a probability space, then $A^c = \Omega \setminus A$ is always also an event.

8) True or False? For every $(\Omega, \mathcal{A})$, where $\mathcal{A}$ is σ-algebra, there is a probability measure P making $(\Omega, \mathcal{A}, \mathrm{P})$ a probability space

9) True or False? $\mathrm{Cov}[\lambda X, \lambda Y] = \lambda^2 \mathrm{Cov}[X, Y]$.

10) True or False? If X^n, Y^n are uncorrelated for all integers $n \geq 0$, then they are independent.

Problem 5) One word answer questions.

1) A Π-system $\mathcal{I}$ has the property that if A, B are in $\mathcal{I}$ then the [ ] is in $\mathcal{I}$.

2) The Lebesgue integral uses the space $\mathcal{S}$ of [ ] for defining the expectation of random variables in $\mathcal{L}^1$ via $E[X] = \mathrm{E}[X^+] - \mathrm{E}[X^-]$.

3) If $(\Omega, \mathcal{A}, \mathrm{P})$ is a probability space, then Ω plays the role of the [ ].

4) A σ algebra for which every event has probability 0 or 1 is called [ ]

5) The smallest Λ-system that contains a Π-system must be a [ ].

6) Simplify $A + A^2 + A^3 + \cdots + A^{20}$. [ ]

7) The doubling strategy in a Casino is called the [ ] strategy in probability theory.

8) Let A be an event in a σ-algebra. What is $A + A^c + A \cdot A \cdot A$? [ ]

9) If A, B both have the same positive probability, then $\mathrm{P}[B|A]/\mathrm{P}[A|B] =$ [ ].

10) If $A = B$ then $\mathrm{P}[A|B] =$ [ ].

"I affirm my awareness of the standards of the Harvard College Honor Code."

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