Lecture 32: Interacting Population Models

Systems

33.1. The two fundamental models for population dynamics of one species are the **Malthus system** of 1798

$$x' = rx$$

modeling unconstrained exponential growth or decay and the **Verhulst system** from 1838

$$x' = r(x - \frac{x}{K}) \ .$$

which has the carry capacity K as an equilibrium.

33.2. When mixing two species, we have not only need to choose the model for each species, we also have to decide **how they interact**. Here is a famous example which has been used for at least a century and with the given numbers for many decades here at Harvard. It is the **Murray system**. A coupled system of logistic equations

$$\begin{aligned} x'(t) &= x(6-2x) - xy \\ y'(t) &= y(4-y) - xy \end{aligned}$$

33.3. Now, when interpreting the system, we have to be able to see how the relation is. There are three different type of coexistence: **Predator-Prey**, **Competition** or **Symbiosis**.

Nullclines

33.4. We have seen nullclines already when looking at **slope fields**. The x-nullclines are curves where $f = 0$, the y-nullclines are curves where $g = 0$.

33.5. For understanding the system it is good to $\boxed{\text{factor}}$:

$$\begin{aligned} x'(t) &= x(6-2x-y) \\ y'(t) &= y(4-y-x) \end{aligned}$$

33.6. $\boxed{\text{Nullclines}}$: Draw the x-nullclines, the place where $x' = 0$ and the y-nullclines, the place where $y' = 0$.

33.7. $\boxed{\text{Equilibria}}$: The equilibrium points are where x-nullclines and y-nullclines intersect.

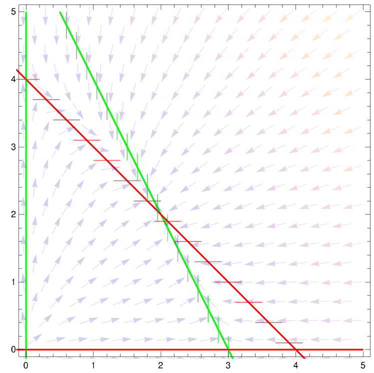

Figure 1. Drawing the nullclines. Color them or mark them differently so that you know which one is the x-nullcline and which is the y-nullcline.

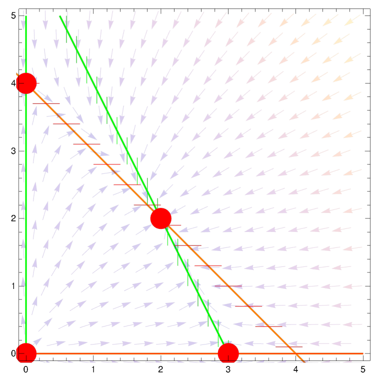

Figure 2. Equilibrium points.

33.8. Complete: We know how the orbits move on the nullclines. Complete the picture by filling in the void:

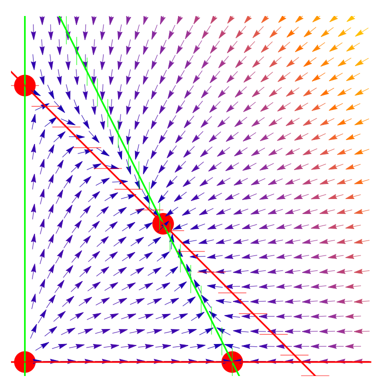

Figure 3. The completed phase space picture.

33.9. We have now a complete picture and know that if we start with a population initial condition $x > 0$ and $y > 0$, then we end up at the equilibrium point $(2, 2)$.

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