CALCULUS AND DIFFERENTIAL EQUATIONS

MATH 1B

Lecture 26: Slope Fields

Review

26.1. Last time we have seen examples of differential equations
A) $\boxed{y'(t) = ky(t)}$. The general solution was $\boxed{y(t) = Ce^{kt}}$.
B) $\boxed{y'(t) = f(t)}$. The anti-derivative $\boxed{\int_0^t f(s)\, ds + C}$ was the general solution.

Slope Fields

26.2. Given a differential equation $\boxed{y' = F(t, y)}$, where the right hand side can depend both on time and on y, we can draw the **slope field** in the $t - y$ plane. Draw at every point a short line with slope $f(t, y)$. Given an initial point (a, b) we can draw the **curve** $y(t)$ which goes through the point (a, b) such that $y'(t)$ is the slope at the point (t, y).

Example: a) Draw the slope field for $y'(t) = y(t)$.

Example: b) Draw the slope field for $y'(t) = \sin(t)$.

Example: c) Draw the slope field for $y'(t) = y(t)(2 - y(t))$ (rumors!)

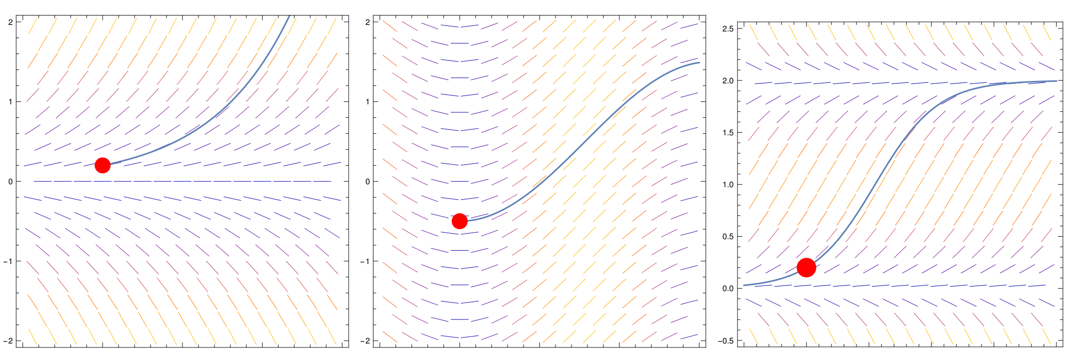

FIGURE 1. Slope fields to the three examples above. We see a solution.

Euler's method

26.3. In order to get a **numerical solution** of a differential equation $y'(t) = F(t, y)$ with $y(0) = y_0$ given, we can chose a time step h, then use

$$y_1 = y_0 + F(0, y_0)h$$

to get an estimate for $y(h)$. This can be repeated

$$y_2 = y_1 + F(h, y_1)h \ .$$

$$y_3 = y_2 + F(2h, y_2)h \ .$$

and so on. This numerical integration method produces a sequence of points $y_0, y_1, y_2, ...$ approximating $(y(0), y(h), y(2h), \dots$ is called the **Euler method**. The results get better if the **step size** h is smaller.

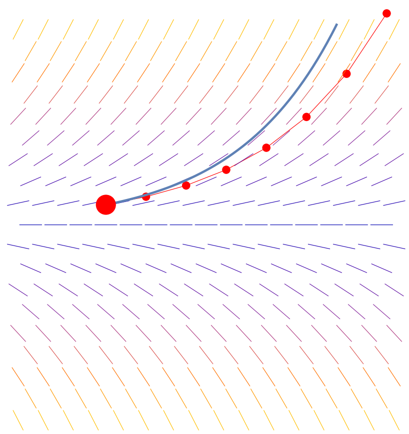

Figure 2. Numerical solution of the differential equation. The Euler method produces an approximation to $y(t)$.

Figure 3. Leonhard Euler (1707-1783)