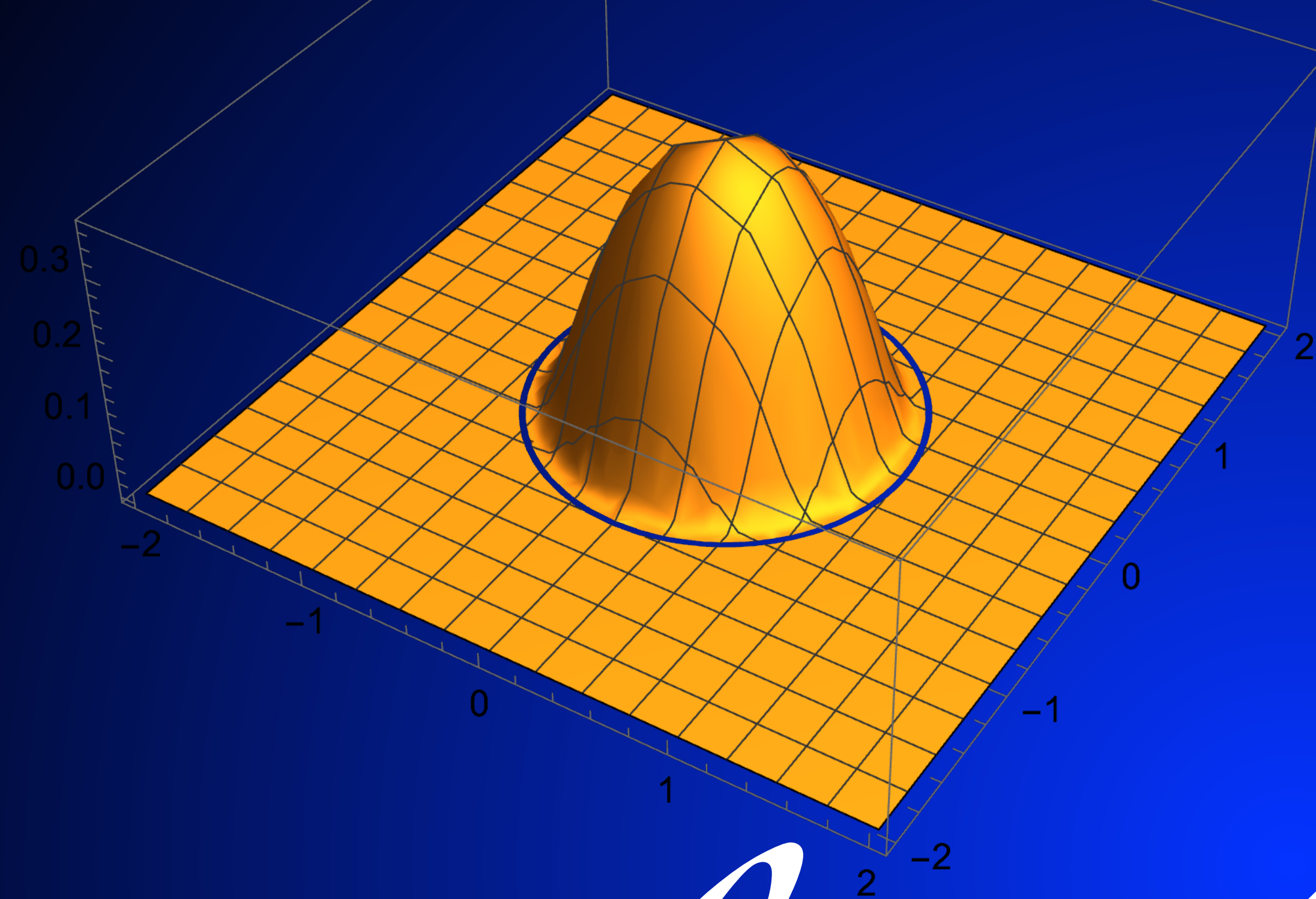




## UNIT 21

# *Island Math*

MATH 22B, MARCH 11, 2022

# *Area and Perimeter*

Which island of 114 acres  
has smallest perimeter?



Which island of 114 acres  
has largest perimeter?



Which island of 2 miles perimeter has  
largest area?

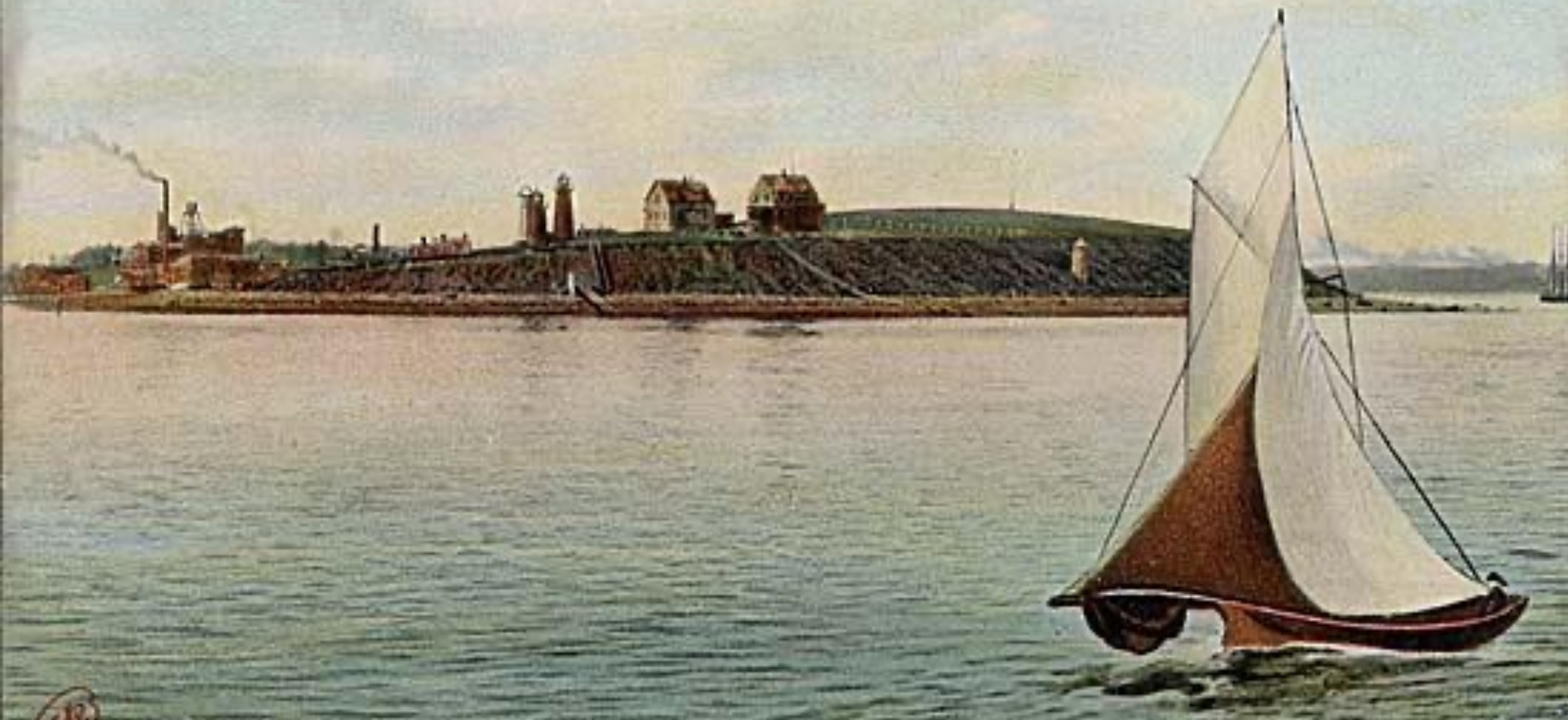


Which island of 2 miles circumference has  
smallest area?



*Spectacle Island*

*Spectacle Island Range Lights, Boston Harbor.*







# *Triangle Problem*

Which triangle of  
fixed circumference  
has maximal area?

# Lagrange Code

```
f = y;  
g = 2+Sqrt[(x-1)^2+y^2]+  
Sqrt[(x+1)^2+y^2];  
Solve[{D[f, x] == L*D[g, x],  
D[f, y] == L*D[g, y], g ==  
6}, {x, y, L}]
```

*Hand Movies*

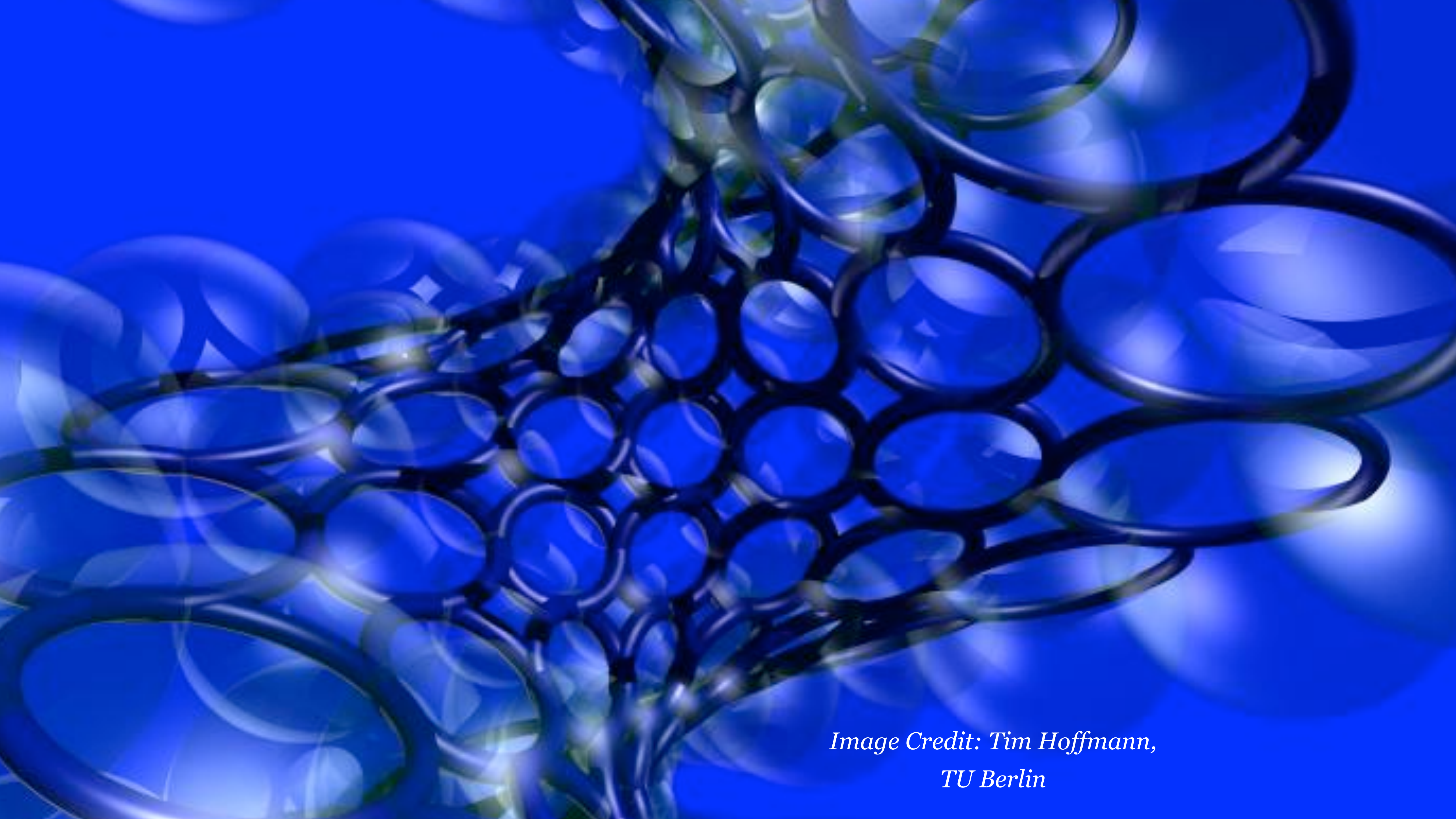


# *Minimal Surfaces*







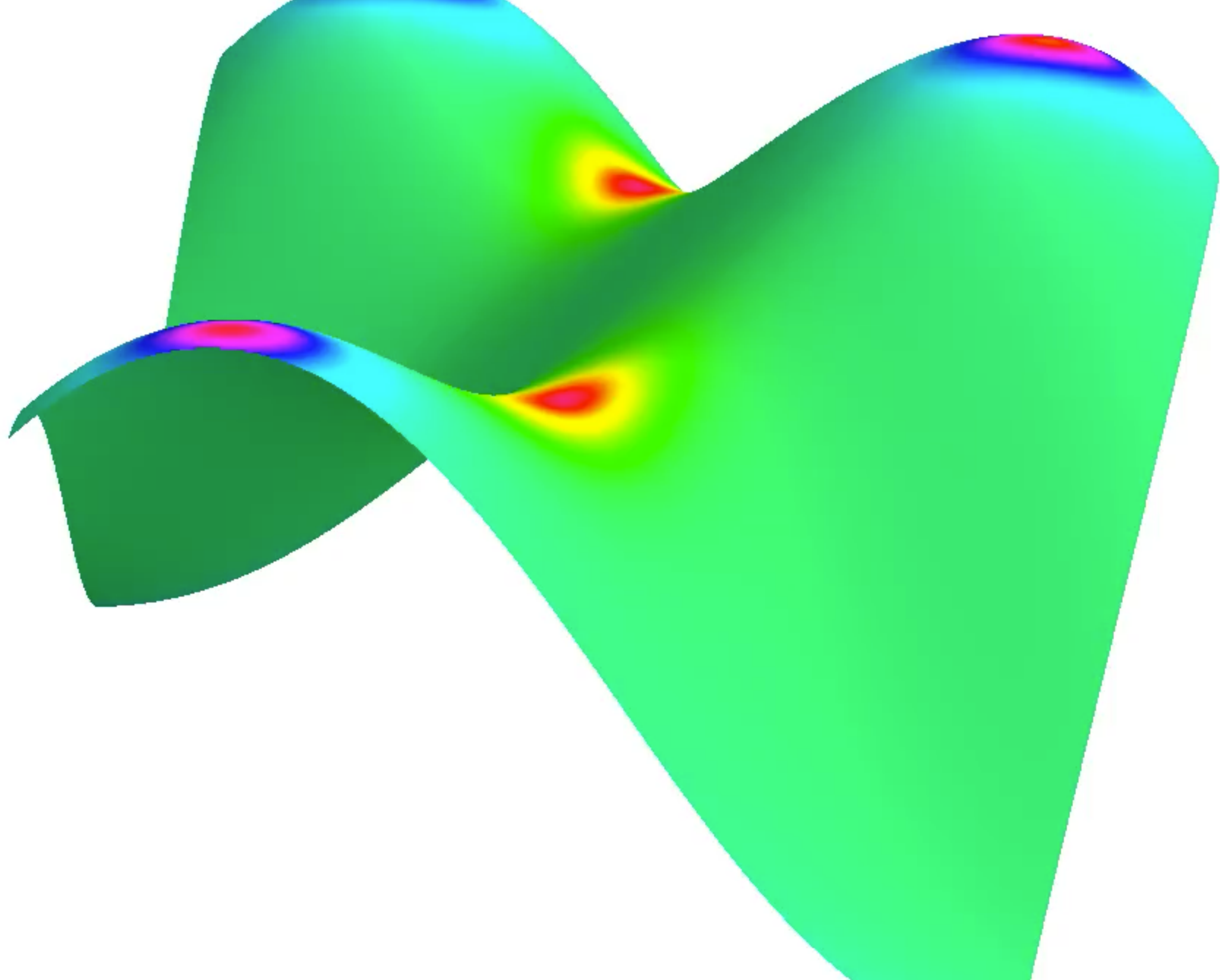


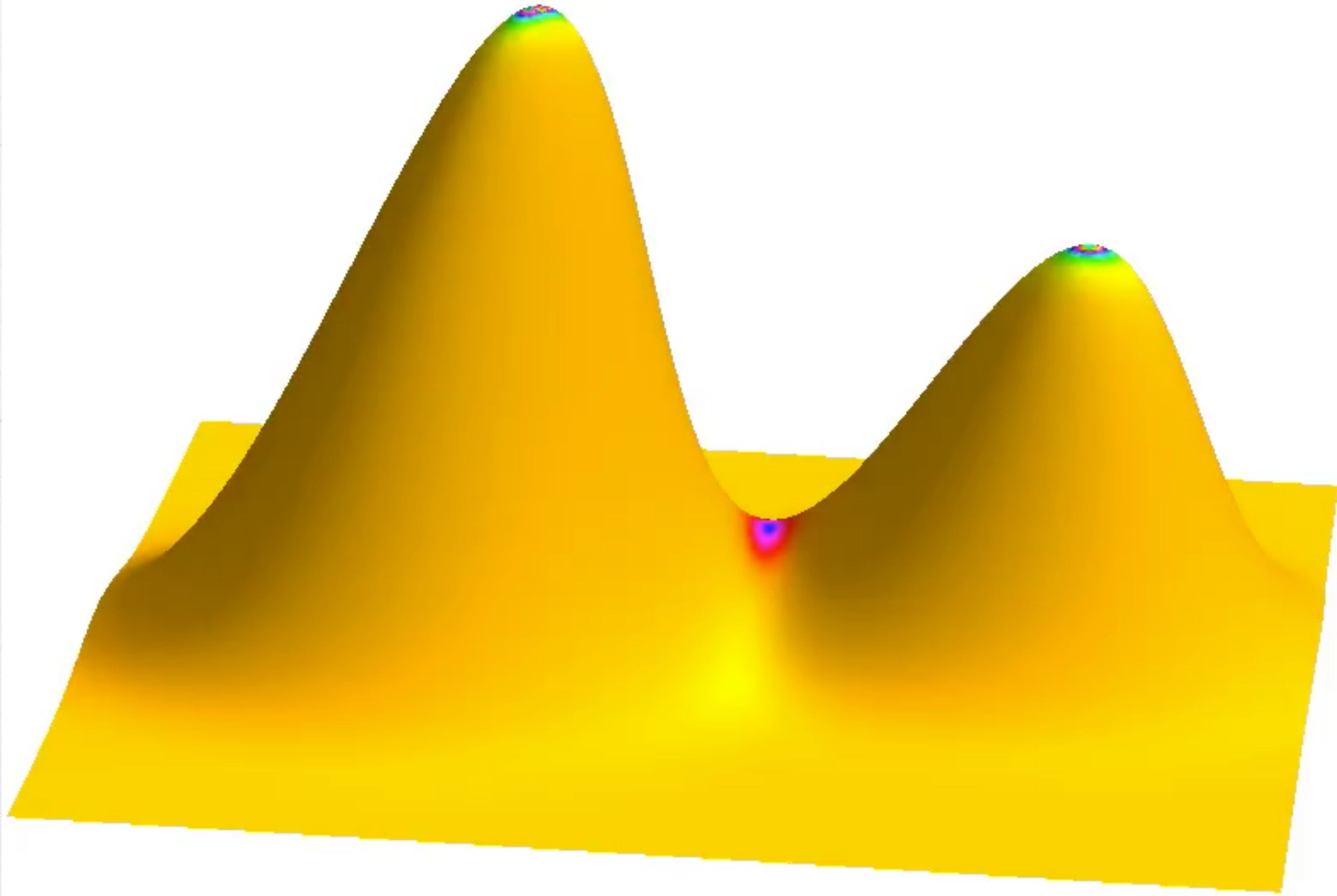
*Image Credit: Tim Hoffmann,  
TU Berlin*

*Mountain peaks, sinks and  
mountain passes*



*Island Theorem*





*Poincare Hopf*

# Henry Poincare

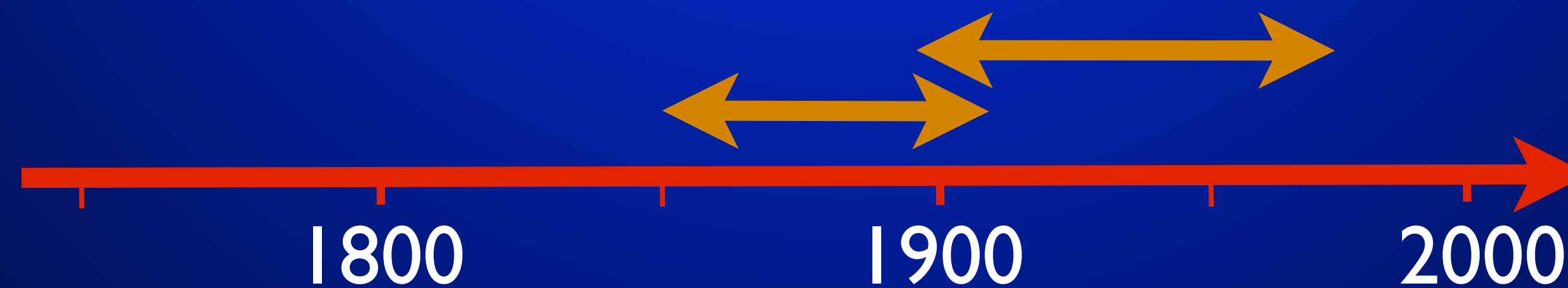


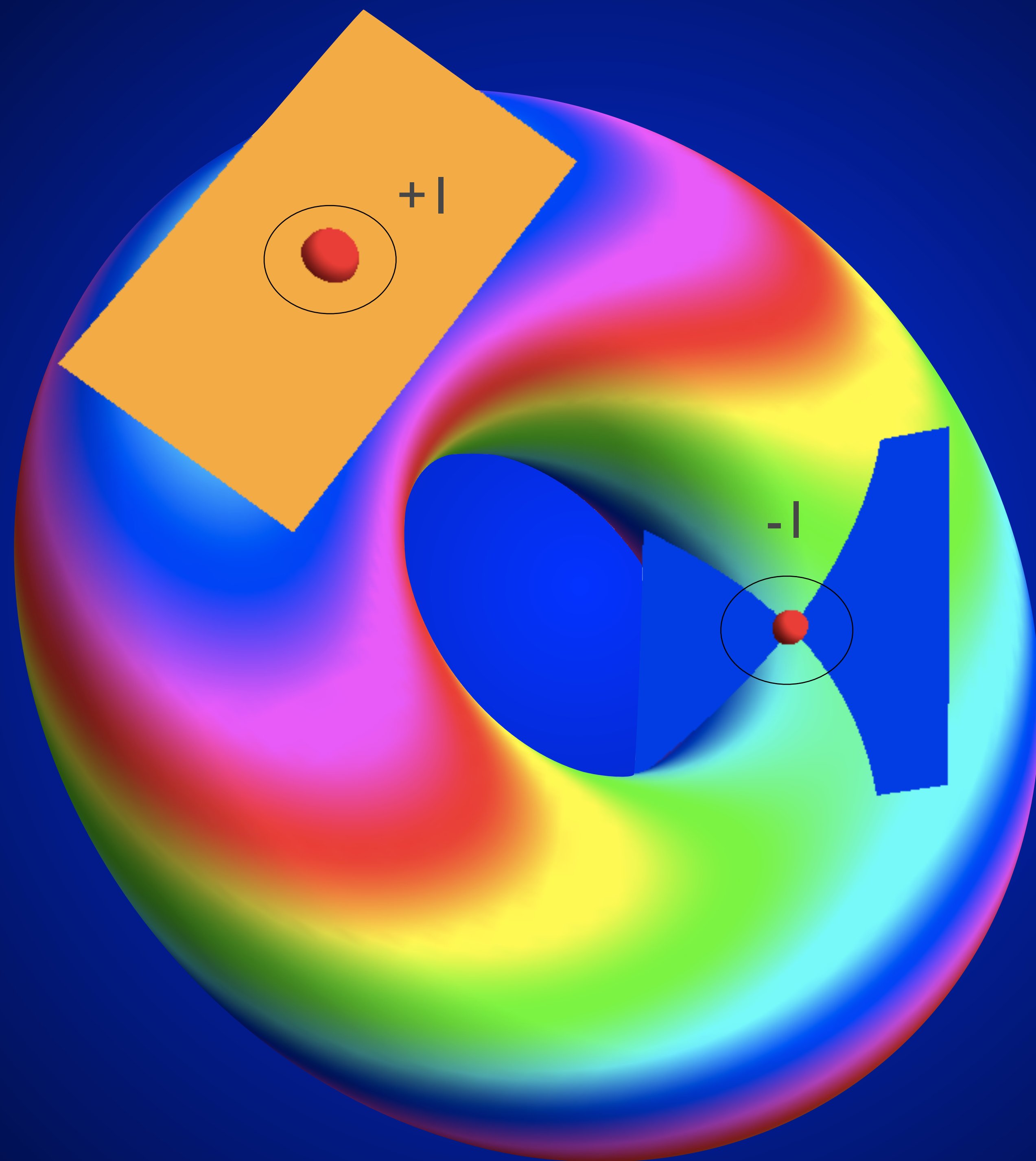
1854-1912

# Heinz Hopf



1894-1971



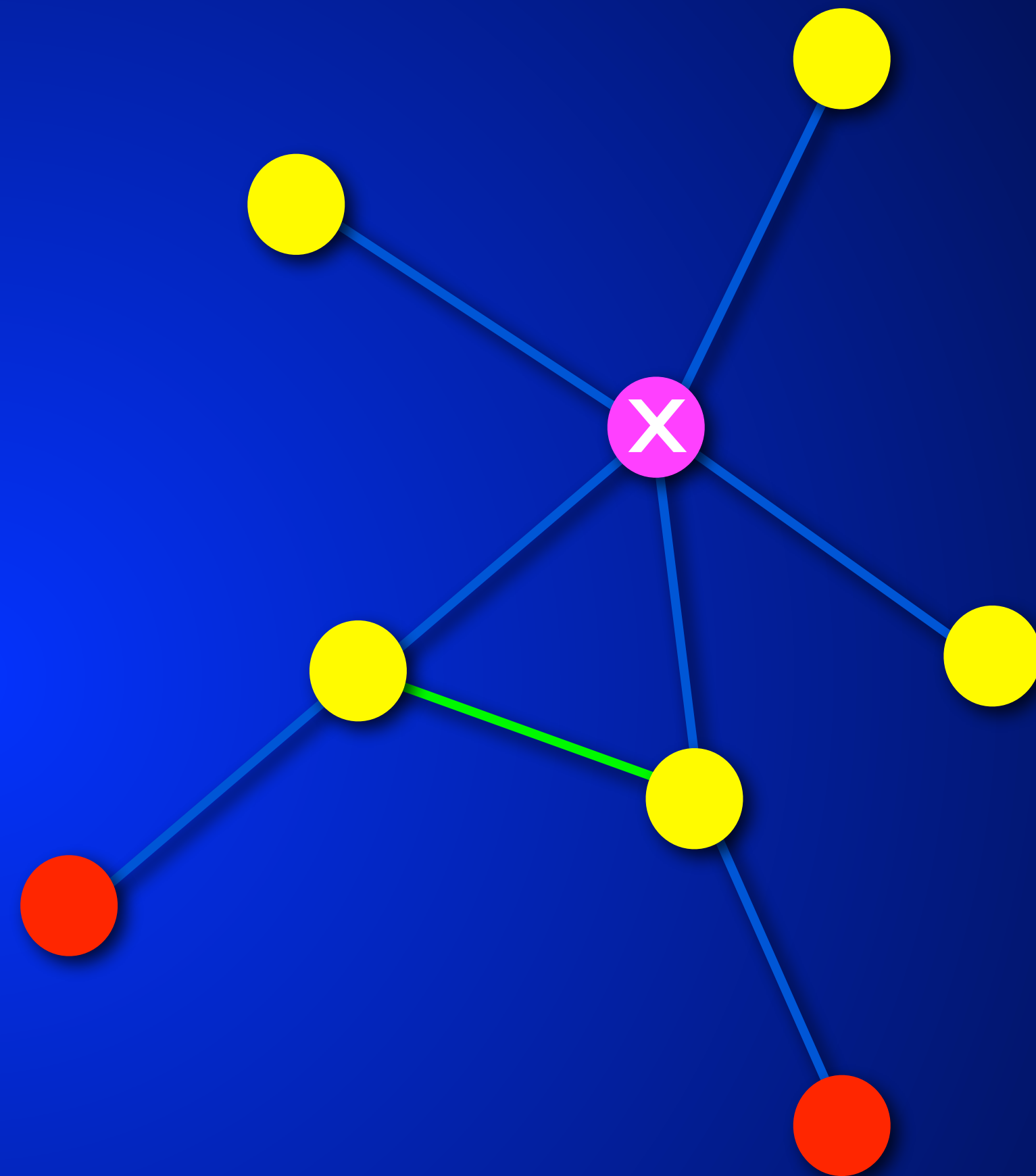
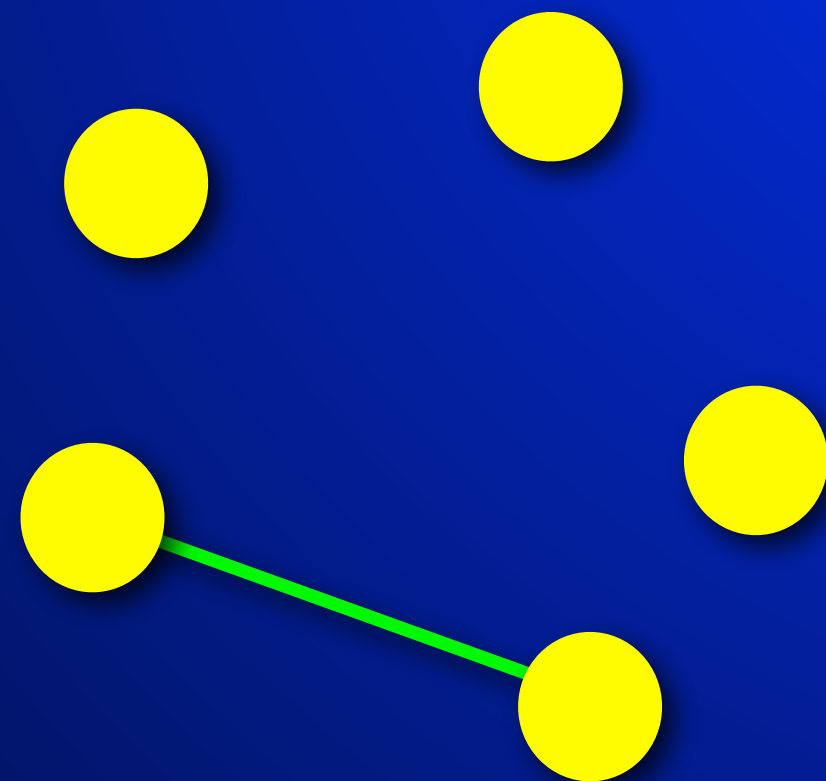


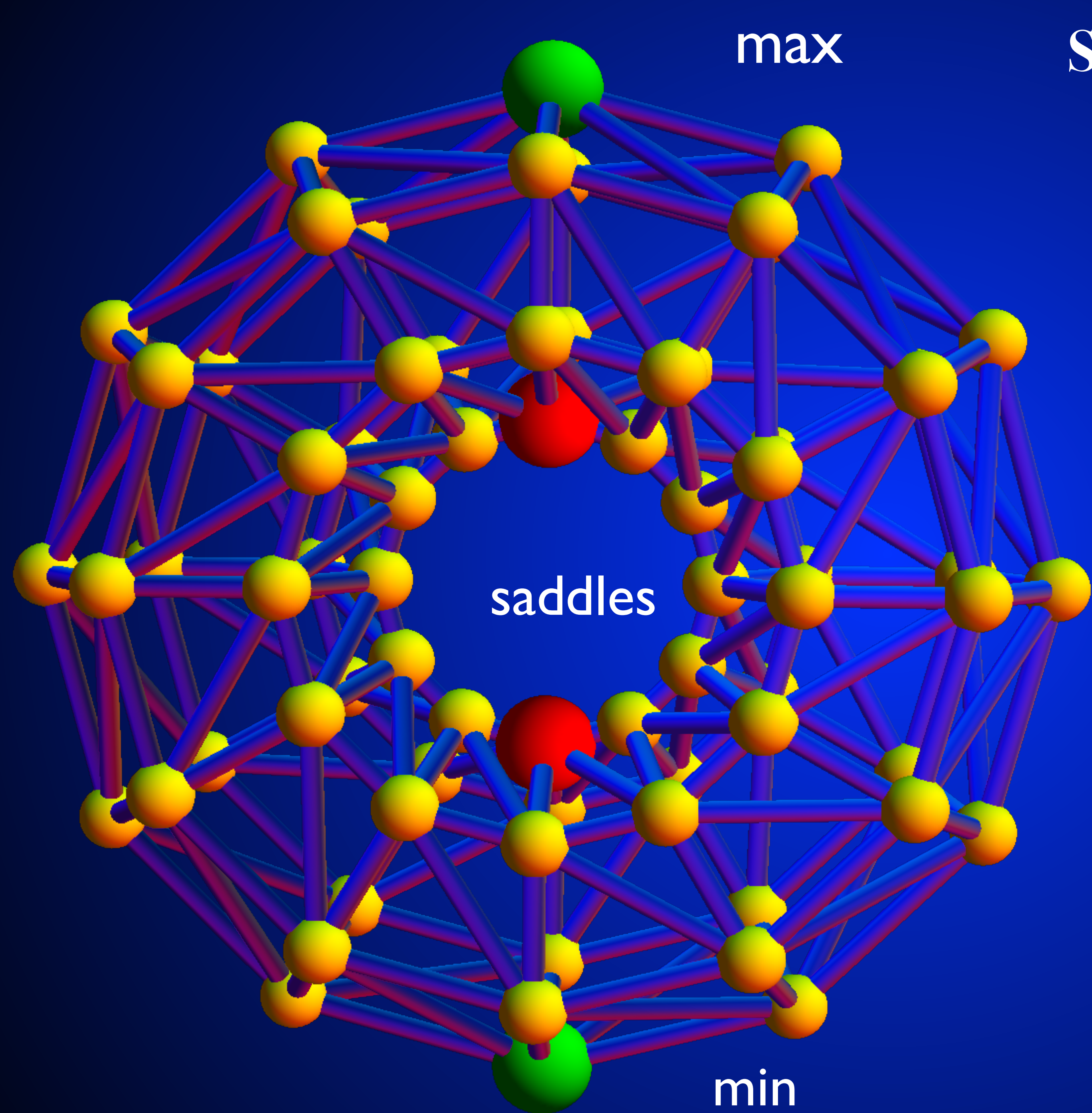
*works also in the*

*Discrete:*

# Unit Sphere

$S(\mathbf{x}) =$





$$\bar{S}(\mathbf{x}) = \mathbf{C}_8$$

$$\mathbf{I} - \chi(\bar{S}(\mathbf{x})) = \mathbf{I}$$

$$\mathbf{I} - \chi(\bar{S}(\mathbf{x})) = -\mathbf{I}$$

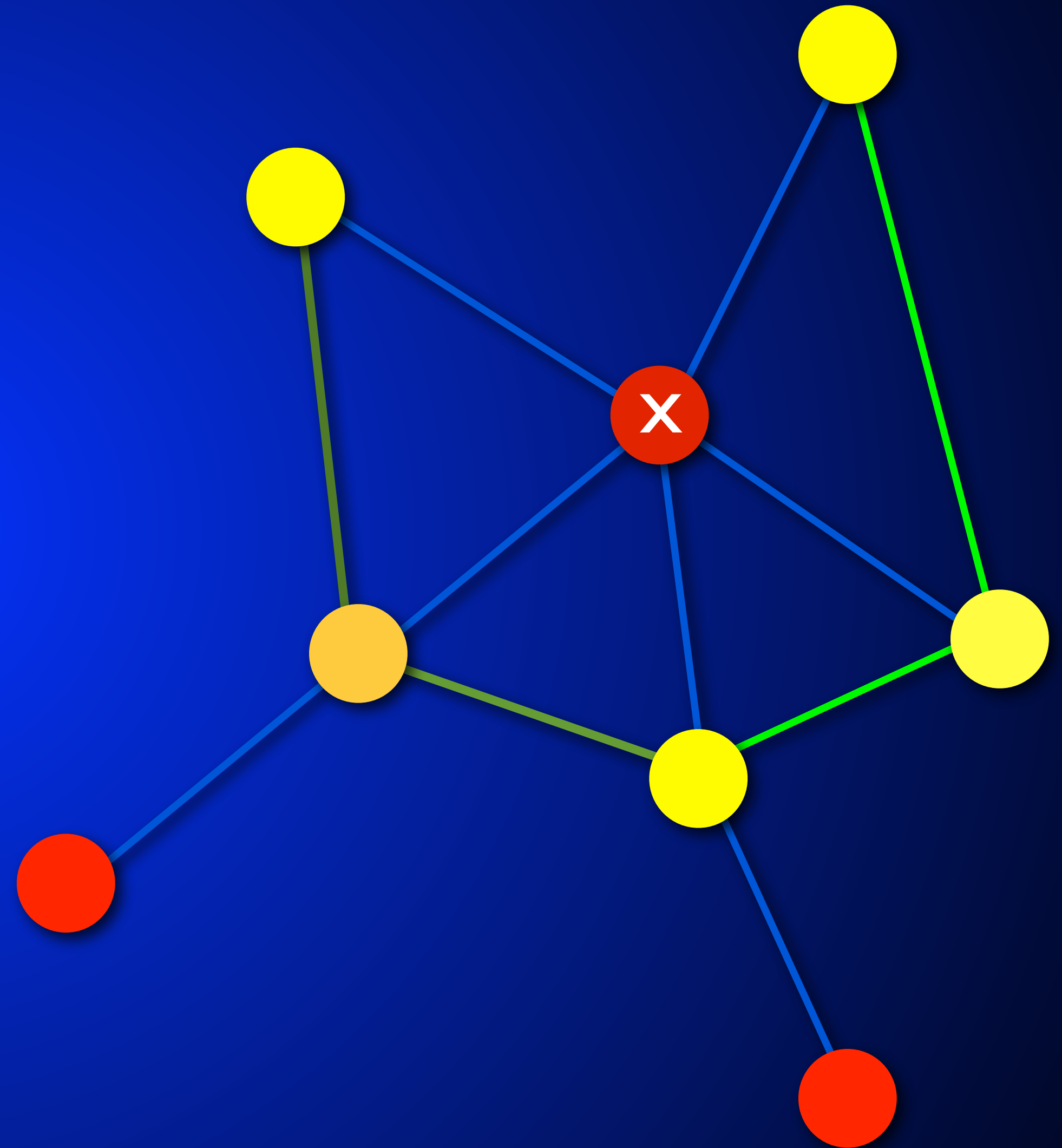
$$\mathbf{I} - \chi(\bar{S}(\mathbf{x})) = \mathbf{I}$$

$$\bar{S}(\mathbf{x}) = \{\}$$

# *Index*

$$S^-(\mathbf{x}) = \{ \mathbf{y} \in S(\mathbf{x}) \mid f(\mathbf{y}) < f(\mathbf{x}) \}$$

$$i(x) = 1 - \chi(S^-(x))$$



*The End*