WIND TOMOGRAPHY IN BINARY SYSTEMS

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ABSTRACT

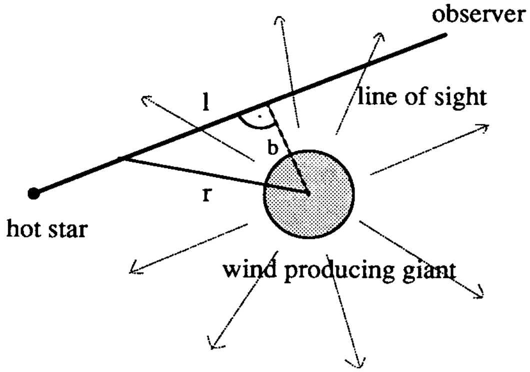
We solve Abel's integral equation by an explicit diagonalization of Abel's operator. This new method is particularly suitable for determining the velocity laws of stellar winds.

1. WIND TOMOGRAPHY AND ABEL'S INTEGRAL

Binary systems in which a compact, point-like radiation source shines through the wind of a companion star offer excellent laboratories for experimentally measuring the run of the velocity profile of stellar winds. In reality, such a situation occurs e.g. in symbiotic binaries (Vogel 1991). In the Figure we depict the geometry of the problem. As a function of the orbital phase, the line of sight from the compact star to the observer passes the extended atmosphere of the companion at different impact parameters, b. The amount of attenuated light is related to the column density $n(b)$ of scatterers or absorbers in the wind along a line of sight depending on the impact parameter b. The observation of different column densities as a function of phase thus provides a "tomographic" view of the stellar wind. Assuming a constant mass loss rate $\dot{M}$, the column density can be expressed via the continuity equation as :

$$n(b) = A(g)(b) = a \int_b^\infty \frac{dr}{rv(r)\sqrt{r^2 - b^2}} = \int_b^\infty \frac{1}{\sqrt{r^2 - b^2}} g(r)\, dr \qquad ,$$

where $a = \frac{2\dot{M}}{4\pi\mu m_H}$ depends on the constant mass loss rate $\dot{M}$ and the mean molecular weight $\mu m_{\rm H}$ in the wind, $g(r) = a/(rv(r))$, and A denotes the Abel integral operator.



The problem is to determine the unknown velocity $v(r)$ of the stellar wind from the observed column density as a function of the impact parameter, or equivalentely, as a function of the orbital phase. This is an inverse problem which are notoriously ill–posed in the sense that small perturbations in the observed data function may result in big errors in the solution. The standard solution to such ill-posed inversion problems are smoothing techniques (for an excellent text book on the subject see Craig and Brown 1986).

2. A NEW APPROACH TO ABEL'S EQUATION - DIAGONALIZATION

We have found an explicit diagonalization of Abel's integral operator by the following eigenfunctions (see Knill, Dgani and Vogel 1993 for the detailed derivations):

$$A(\psi_i) = \lambda_i \cdot \psi_i \ , \qquad \text{where} \qquad \psi_i(r) = r^{-i} \ , \qquad\qquad i \geq 1$$

are the eigenfunctions to the eigenvalues

$$\lambda_i = \int_0^{\pi/2} \cos^{i-1}(\phi)\, d\phi = \begin{cases} \frac{(i-2)!!}{(i-1)!!} & , \ i \text{ even} \\ \frac{\pi}{2} \cdot \frac{(i-2)!!}{(i-1)!!} & , \ i \text{ odd} \end{cases}$$

where $i!!$ denotes the double factorial of n which are recursively defined by $0!! = 1!! = 1, i!! = i \cdot (i-2)!!$. We note that these eigenvalues constitute a monotonically decreasing series.

3. THE INVERSION METHOD

Let's assume for the moment that $g = \sum_{i=1}^{\infty} g_i r^{-i}$ is analytic in $[0, \infty)$. The operator A then maps g into the function $Ag = \sum_{i=1}^{\infty} g_i \lambda_i b^{-i}$, and the inversion of A is explicitely given by

$$A^{-1} \ : \qquad \sum_{i=1}^{\infty} n_i b^{-i} \qquad \longmapsto \qquad \sum_{i=1}^{\infty} \frac{n_i}{\lambda_i} r^{-i}$$

This diagonalization again shows that Abel's operator is ill–posed, since it has an unbounded inverse, that means, the data function $n(b) = \sum_{i=1}^{\infty} n_i b^{-i}$ with a perturbation δn, results in an error for the source function g of

$$\delta g(r) = \sum_{i=1}^{\infty} \frac{\delta n_i}{\lambda_i} r^{-i} \qquad ,$$

where $\lambda_i \to 0$ as $i \to \infty$. Thus, δg can be arbitrarily large for very small errors in the data δn. The simplest solution is to truncate the sum. This leads to an accurate and well–posed inversion on polynomials, if the source function $g(r)$ is well approximated by a polynomial of the eigenfunctions.

Stellar winds are expected to approach a terminal constant velocity for large distances. Thus, the source function $g(r) = a/(rv(r))$ asymptotically approaches the first eigenfunction $1/r$. Moreover, it can also be shown that all resonable stellar winds can be well represented by a low order polynomial in $1/r$, namely by the first few eigenfunctions. Thus, our method is well suited for the reconstruction of the velocity profile in stellar winds using the following procedure:

- From the accurate data function $n(b)$, we only know an experimentally determined data set

$$\{(b_i, n_i(b_i)) \mid i = 1, \ldots N,\ 0 < b_i < \infty\} \qquad .$$

- These noisy data is fitted with a polynomial of order K

$$n^{(K)}(b) = \sum_{i=1}^{K} n_i^{(K)} b^{-i} \qquad .$$

- The fitting polynomial $n^{(K)}$ is now transformed by A^{-1} to

$$g^{(K)}(r) = \sum_{i=1}^{K} \frac{n_i^{(K)}}{\lambda_i} r^{-i} \qquad ,$$

  with the λ_i's given above, and from this the velocity law is easily derived.

We have tested our method for different velocity laws representing radiatively driven winds from hot stars, winds from K supergiants, and winds expected from cool, dust driven giants. We found satisfactory results in that the errror in the reconstructed function scales linearily with the error in the data.

Bibliography

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