Problem 3

a) The matrix

$$A = [v_1 v_2 v_3] = \begin{bmatrix} 1 & 1 & -1 \\ 4 & 6 & 2 \\ 5 & 7 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

is one of the possibilities. You can also use $[v_1 v_2]$ once you know that v_3 is a linearcombination of v_1 and v_2.

b) Row-reduced echelon form of A is

$$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore columns with leading 1's are independent and $v_3 = -4v_1 + 3v_2$ is redundant. So the basis is

$$(v_1, v_2) = \left(\begin{bmatrix} 1 \\ 4 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \\ 7 \\ 3 \end{bmatrix} \right)$$

c) Denote

$$x = \begin{bmatrix} -1 \\ 4 \\ 3 \\ 2 \end{bmatrix}$$

Row-reduced echelon form of the augmented matrix

$$\left[\begin{array}{cc|c} 1 & 1 & -1 \\ 4 & 6 & 4 \\ 5 & 7 & 3 \\ 2 & 3 & 2 \end{array}\right]$$

is

$$\left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Therefore there is a linear relation $x = -5v_1 + 4v_2$ and so x is in V.