Anomalies and quiches

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Based on arXiv:2307.08147, arXiv:2209.07471 (w/Greg Moore, Constantin Teleman), and work in progress w/Colleen Delaney, Julia Plavnik and Constantin Teleman

Outline

(1) Anomalies in QM and QFT

(2) Groups of symmetries and anomalies

(3) Algebras of symmetries; quiches

(4) Anomalous algebras of symmetries (ongoing new work)

(5) Zesting of fusion categories (ongoing new work)

(1) Anomalies in quantum theory

Quantum theory is projective; quantization is linear

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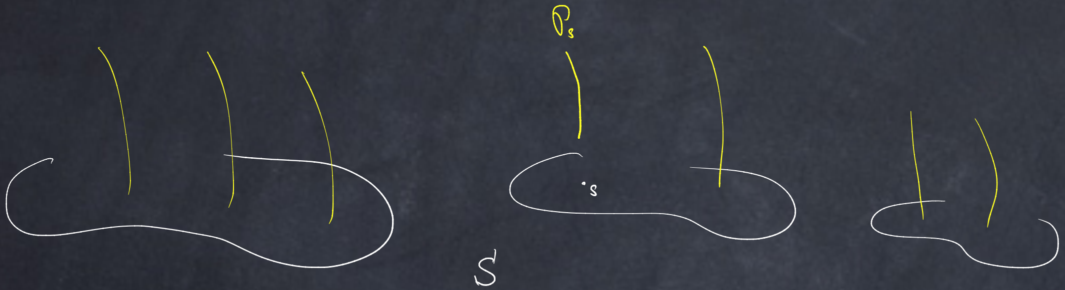
This is more interesting for *families* of QM systems or a QM system with symmetry

A family of QM systems has data a fiber bundle of projective spaces $\mathcal{P} \longrightarrow S$



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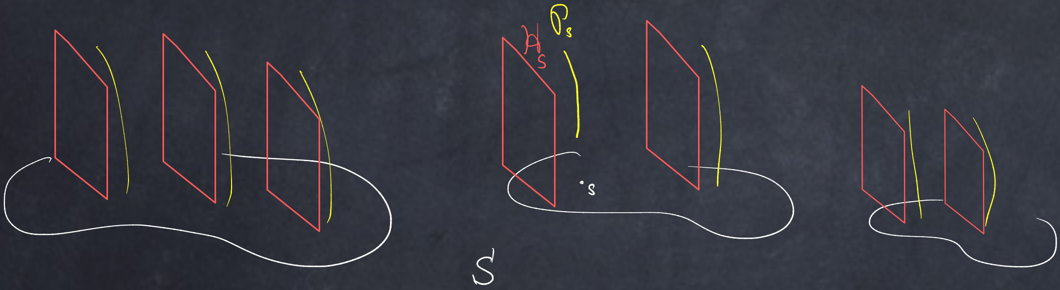

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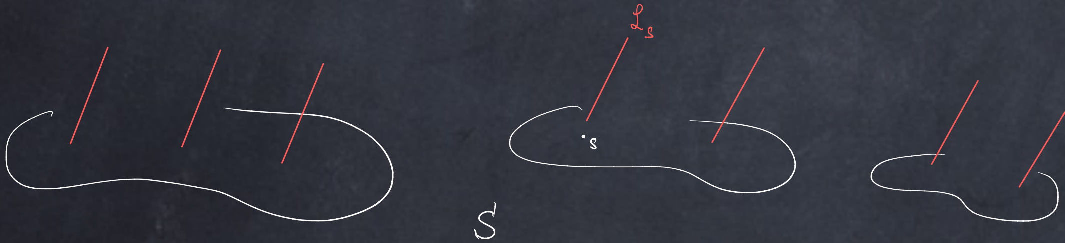
Remark: A line bundle over S is a family of *invertible* QM systems



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$\mathbb{P}$ $\mathcal{H}$

$* \circlearrowleft G$

Ex: $\mathbb{T} \to SU_2 \to SO_3$

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Example: The harmonic oscillator: $G =$ symplectic group and $\widetilde{G} =$ metaplectic group

Wick-rotated QFT as a linear representation

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Man_n	category of smooth n-manifolds and local diffeomorphisms
sSet	category of simplicial sets

Definition: A *Wick-rotated field* is a sheaf

$$\mathcal{F}\colon\ \mathrm{Man}_n^{\mathrm{op}} \longrightarrow \mathrm{sSet}$$

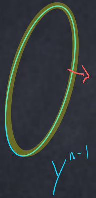
Examples: Riemannian metrics, G-connections, $\mathbb{R}$-valued functions, M-valued functions, orientations, spin structures, gerbes, ...

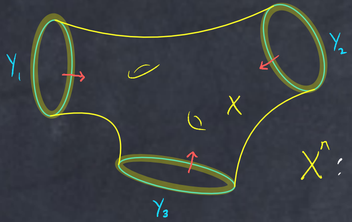
$\mathcal{F}$ can be a *collection* of fields; $\mathcal{F}(M)$ is the simplicial set of fields on an n-manifold M

Axiom System: $\mathrm{Bord}_n(\mathcal{F})$ bordism category

n dimension of spacetime

$\mathcal{F}$ background fields (orientation, Riemannian metric, . . .)





$$X^n : Y_1 \sqcup Y_2 \sqcup Y_3 \to \emptyset^{n-1}$$

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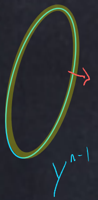
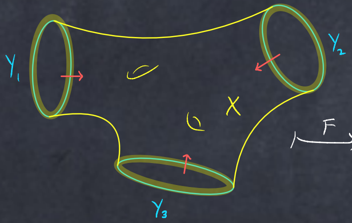
Vect linear category of topological vector spaces and linear maps

$F\colon \mathrm{Bord}_n(\mathcal{F}) \longrightarrow \mathrm{Vect}$ linear representation of bordism category



$\stackrel{F}{\longmapsto} F(Y)$



$X^n : Y_1 \sqcup Y_2 \sqcup Y_3 \to \emptyset^{n-1}$

$\stackrel{F}{\longmapsto} \Big(F(X) : F(Y_1) \otimes F(Y_2) \otimes F(Y_3) \longrightarrow \mathbb{C} \Big)$

Wick-rotated QFT as a projective representation; the anomaly

Proj category of (holomorphic) projective spaces and holomorphic maps

Vect category of topological vector spaces and linear maps

Line category of complex lines and invertible linear maps

$$\mathrm{Line} \longrightarrow \mathrm{Vect} \longrightarrow \mathrm{Proj}$$

Jackson Van Dyke

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Projective theory $\overline{F}$

Its anomaly = projectivity α and resulting extension of the bordism category

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Ratio of trivializations: an invertible n-dimensional theory

Anomaly as an invertible field theory

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$\Sigma(\mathrm{Line})$ is a groupoid of gerbes, a categorification of Line

The anomaly theory α is a *once-categorified* n-dimensional invertible field theory

An n-dimensional $\boxed{\text{theory } \overline{F} \textit{ relative} \text{ to } \alpha}$ assigns $\overline{F}(X^n)\colon \mathbb{C} \longrightarrow \alpha(X^n)$ for X^n closed

To Y^{n-1} closed, $\overline{F}$ assigns a projective space with projectivity $\alpha(Y^{n-1})$

Ratios of trivializations of α: a standard type of n-dimensional invertible theory

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G (finite, Lie, "higher") group of symmetries of F over background fields $\mathcal{F}$

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Couple F to a background gauge field: (i)

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It could happen that α is pulled back from the base; this corresponds to a projective action of G and is the situation we generalize to algebras of symmetries

Example: G finite, α given by a Dijkgraaf-Witten cocycle

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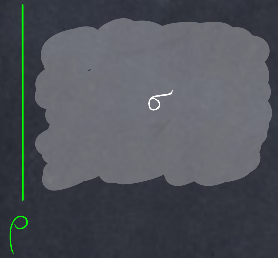

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(iii) use Topological Field Theory technology to compute universal relations among defects

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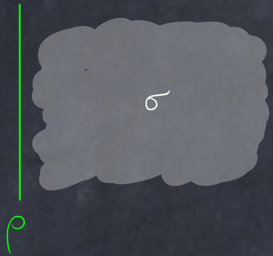
ρ is a topological right *relative* theory (Will Stewart thesis)



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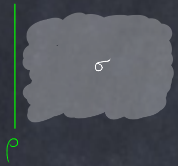


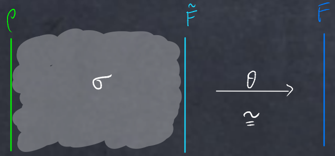
(σ, ρ)-module: F n-dimensional field theory

A (σ, ρ)-*module structure* on F is $(\widetilde{F}, \theta)$ in which

$\widetilde{F}$ is a (nontopological) left relative theory for σ

θ is an isomorphism $\theta\colon \rho \otimes_{\sigma} \widetilde{F} \xrightarrow{\cong} F$ (sandwich presentation)

$$\rho \otimes_{\sigma} \tilde{F} = \langle \rho \,|\, \sigma \,|\, \tilde{F} \rangle$$



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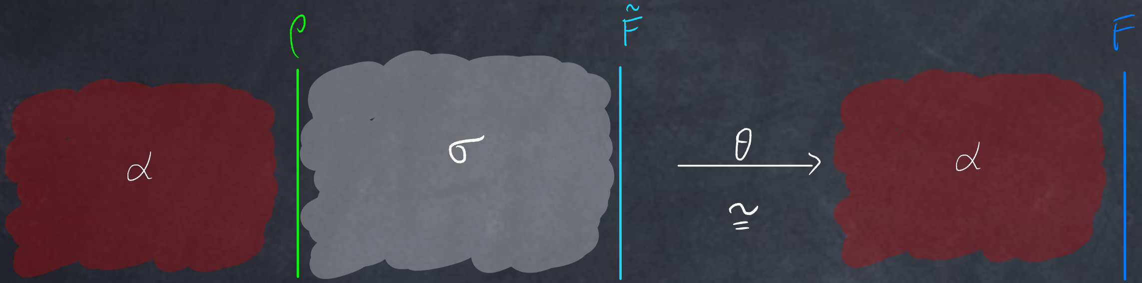

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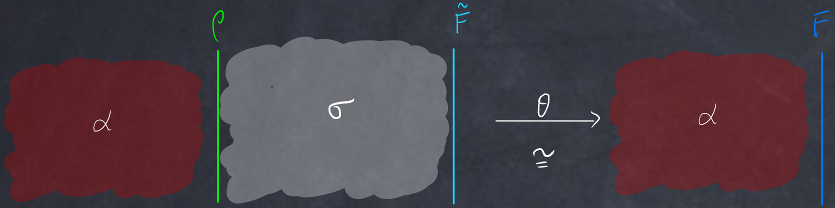

There is a useful class of TFT's called *finite homotopy theories*, which are constructed by finite quantization using π-finite spaces

Example: For a finite (higher) group G—with or without a cocycle—σ is the associated finite gauge theory, and it extends to a full $(n+1)$-dimensional TFT; the corresponding π-finite space is BG

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We now come to the central issue addressed in the new work

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- We use extra structure on a quiche to express anomalous symmetries
- There is a geometric and an algebraic version of the extra structure
- "Anomaly" is a relative notion in this context, not an absolute notion

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Disclaimers:

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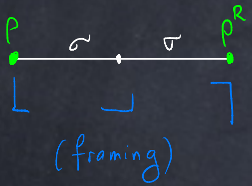

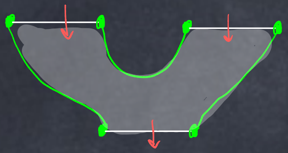

multiplication

unit

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- If $\sigma(\mathrm{pt})$ is an algebra, then the *regular module* is Dirichlet
- If a path connected π-finite space $\mathcal{X}$ defines a finite homotopy TFT, then a basepoint in $\mathcal{X}$ determines a Dirichlet boundary theory

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Remark: We freely move between right and left boundary theories by taking adjoints

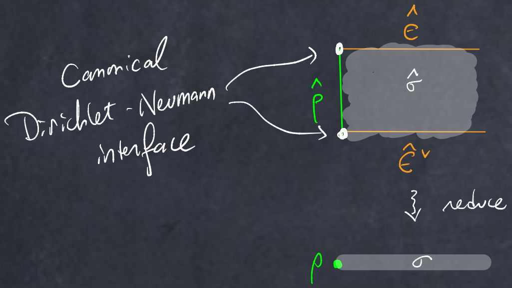
Geometric picture

(σ, ρ) n-dimensional quiche; for convenience, suppose σ is a full $(n+1)$-dimensional theory

Extra hypothesis: (σ, ρ) is the dimensional reduction of an $(n+1)$-dimensional quiche $(\hat{\sigma}, \hat{\rho})$ with respect to a Neumann boundary $\hat{\epsilon}$ and its dual $\hat{\epsilon}^\vee$:

Dirichlet

$$(\sigma, \rho) \cong \langle \hat{\epsilon} | (\hat{\sigma}, \hat{\rho}) | \hat{\epsilon}^\vee \rangle$$



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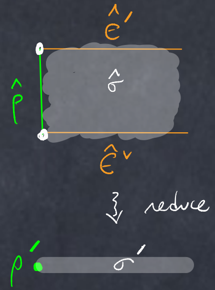

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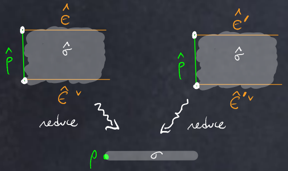

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Example: G finite (higher group), σ, $\hat{\sigma}$ finite gauge theories, $\hat{\epsilon}$ usual Neumann, $\hat{\epsilon}'$ Neumann with DW cocycle

Geometric picture: variation

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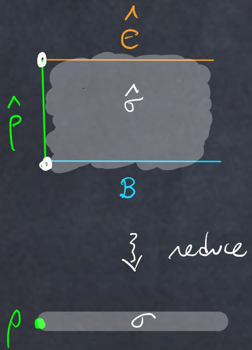

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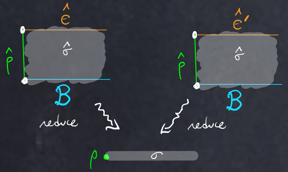

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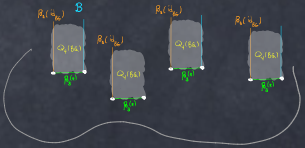

Zesting: We encounter a *parametrized family* with σ, $\hat{\sigma}$ finite gauge theories, $\hat{\epsilon}$, $\hat{\epsilon}'$ usual Neumann, but in the family $\hat{\epsilon}'$ is twisted with respect to $\hat{\epsilon}$

Algebraic picture

There is a corresponding algebraic picture based on the following idea: A “twisted version” of an algebra A is an invertible A-bimodule M (Example: $A = C(X)$ is an algebra of functions on a space X and M is the bimodule of sections of a line bundle $L \longrightarrow X$)

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The extra structure on a quiche (σ, ρ) is an algebra structure (in the higher category of quiche), and then a twisted form is an invertible (σ, ρ)-bimodule

(5) Zesting of fusion categories (ongoing new work)

$\Phi = G \ltimes \Phi_1$ "cross product" of a finite group G and a fusion category Φ_1:

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Interpret as a G-action on Φ_1—a fibering of Φ_1 over BG—and so express Φ as a quotient (gauging) of Φ_1 by G, best in terms of the corresponding 3d *Turaev-Viro* theories $\mathcal{T}_3(\Phi), \mathcal{T}_3(\Phi_1)$ and the 4d finite gauge theory $\mathcal{G}_4(BG)$ with augmentation (Neumann bdy) ϵ:

$$\mathcal{T}_3(\Phi) = \epsilon \otimes_{\mathcal{G}_4(BG)} \underset{\alpha}{\mathcal{T}_3(\Phi_1)}$$

The homotopy groups of $\mathrm{BrPic}(\Phi_1)$ are expressed in terms of the Drinfeld center $Z\Phi_1$

$$\begin{array}{cc} \pi_2 & \mathbb{C}^\times \\ \pi_1 & A \\ \pi_0 & \pi_0\,\mathrm{Aut}(Z\Phi_1) \end{array} \qquad \left.\begin{array}{c}\mathbb{C}^\times \\ A\end{array}\right\} \mathcal{I}$$

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The data of $\alpha' - \alpha$ is a cocycle $\lambda \in Z^2(BG; A)$ and a cochain $\nu \in C^3(BG; \mathbb{C}^\times)$

Set $\pi_0 = \pi_0 \operatorname{Aut}(Z\Phi_1)$, so the Postnikov tower of $X = B\big(\operatorname{BrPic}(\Phi_1)\big)$ is:

$$
\begin{tikzcd}
 & & \mathcal{E} \arrow[dl, "\Phi_1"'] \\
B^3\mathbb{C}^\times \arrow[r] & X \arrow[d] & \\
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& BG \arrow[r] \arrow[u, bend left, "f_\alpha"] \arrow[rrruu, bend right=15, "B\alpha"] \arrow[rrru, bend right=15] & B^3A & & B\pi_0 \arrow[r] & B^3A
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The lift also induces a section $f_\alpha\colon BG \longrightarrow \mathcal{X}$

$(\mathcal{X}, \mu)$	π-finite space $\mathcal{X}$ with cocycle μ
$f : \mathcal{Y} \longrightarrow \mathcal{X}$	π-finite map
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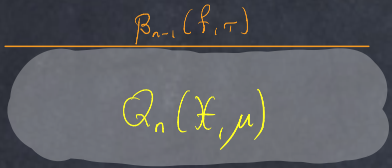

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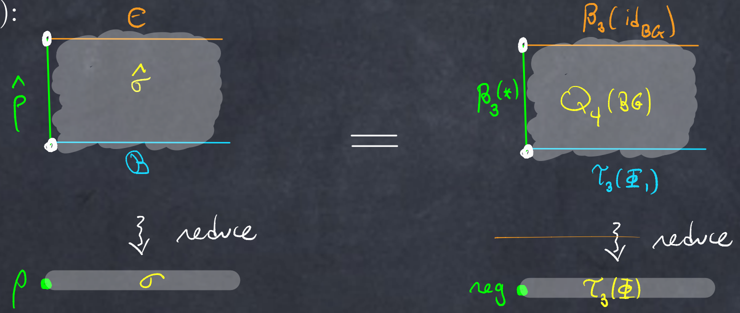

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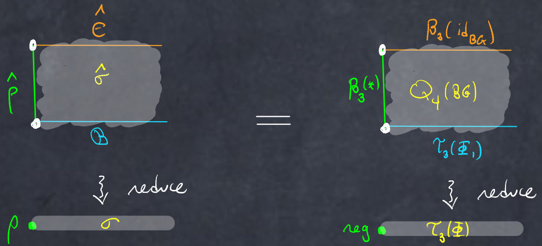

For zesting purposes we replace gauge theory $Q_4(BG)$ with $Q_4(\mathcal{X}, \mu)$ ...

$$
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 & & & B^3\mathbb{C}^\times \arrow[r] & X \arrow[d] & \\
B^2A \arrow[r] & \mathcal{X} \arrow[r, "\mu"] \arrow[d] & B^4\mathbb{C}^\times & B^2A \arrow[r] & X_2 \arrow[r] \arrow[d] & B^4\mathbb{C}^\times \\
 & BG \arrow[r] \arrow[uurrr, bend right, "B\alpha"] \arrow[urrr, bend right] & B^3A & & B\pi_0 \arrow[r] & B^3A
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$$
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B^2A \arrow[r] & \mathcal{X} \arrow[r, "\mu"] \arrow[d] & B^4\mathbb{C}^\times & & B^2A \arrow[r] & X_2 \arrow[r] \arrow[d] & B^4\mathbb{C}^\times \\
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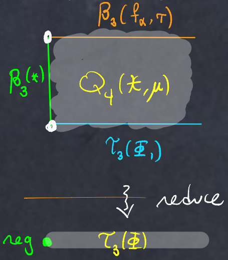

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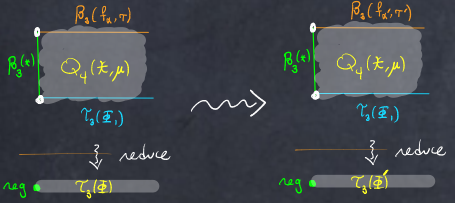

Zesting is the replacement $\alpha \rightsquigarrow \alpha'$ and so $(f_\alpha, \tau) \rightsquigarrow (f_{\alpha'}, \tau')$

Zesting and anomalies

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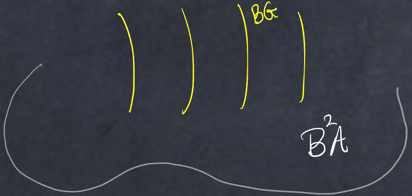

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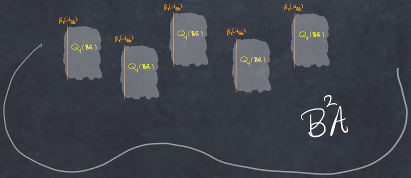

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Zesting: change of a family of Neumann conditions: