**Homework 12**
Real Analysis
Math 212a – Harvard University – Fall 1998
Due Friday, 11 December 1998

Royden: Chapter 10 (27 (see 10-12), 28, 40, 41, 48, 53, 54, 55).

1. Show that the closed unit ball in a Banach space $X$ is compact iff $X$ is finite-dimensional.

2. Show that the set of extreme points of a compact convex set in $\mathbb{R}^2$ is always closed.

3. Give an example of a compact convex set in $\mathbb{R}^3$ whose extreme points are not closed.

4. Show that for $1 \leq p < \infty$,

$$N^{-1} \sum_{1}^{N} \sin(nx) \to 0$$

in $L^p[0, 2\pi]$ as $N \to \infty$.

5. What happens for $p = \infty$?