Topology

In–class Midterm, 24 October 2013

Name ____

Aim for clear, concise, complete answers. All problems have equal weight.

1. Mark each of the following assertions True (T) or False (F).

(a) ☐ Every function from a finite set to itself must be 1-to-1.

(b) ☐ If $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function and $\sup |f'(x)| < \infty$, then f is uniformly continuous.

(c) ☐ The set $\mathcal{F} \subset C([0,1])$ consisting of all differentiable functions f such that $|f(x)| \leq 1$ and $|f'(x)| \leq 1$ is compact.

(d) ☐ The space $\mathbb{R}_\ell$, the real numbers with the lower limit topology, is Hausdorff.

(e) ☐ In a Hausdorff space, any finite set is closed.

(f) ☐ There exists a countable sequence of open sets $U_i \subset \mathbb{R}$ such that $\bigcap_1^\infty U_i = \mathbb{R} - \mathbb{Q}$.

(g) ☐ Any sequence in a totally bounded metric space has a Cauchy subsequence.

(h) ☐ In a compact topological space, any infinite set has a limit point.

(i) ☐ If f is continuous, then $\overline{f^{-1}(A)} = f^{-1}(\overline{A})$.

(j) ☐ The space $[0,1]^{\mathbb{R}}$ in the product topology is normal.

2. Define the following terms. (i) Cauchy sequence. (ii) Normal topological space. (iii) Equicontinuous family of functions. (iv) Basis for a topology.

3. Let X be a path-connected topological space. Show that $X^{\mathbb{N}}$, in the product topology, is also path-connected.

4. Let $X_1 \supset X_2 \supset X_3 \ldots$ be a nested sequence of closed, connected subsets of a compact Hausdorff space Y. Show that $X = \bigcap X_i$ is connected. BONUS: Show for $Y = \mathbb{R}^2$, X need not be connected.