Math Xb Spring 2004
Worksheet: Geometric Sums
February 20, 2004

Solutions

For each of the following sums, determine if the sum is geometric. If the sum is geometric, determine the common ratio of the sum and the number of terms in the sum. Also express the sum in closed form.

1. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}$

Geometric

common ratio = $1/2$

terms = 5

closed form = $\dfrac{\frac{1}{2}-\frac{1}{64}}{1-\frac{1}{2}} = \dfrac{31}{32} = 0.96875$

2. $1-5+25-125+\cdots+5^{12}$ = $1 + \underbrace{(-5)+(-5)^2+(-5)^3+\ldots+(-5)^{12}}_{12 \text{ terms}}$

Geometric

common ratio = -5

terms = 13

closed form = $\dfrac{1+5^{13}}{1+5} = 203{,}450{,}521$

3. $1+3+5+7+9+11$

Not Geometric

4. $\frac{3}{4}+\frac{1}{2}+\frac{1}{3}+\frac{2}{9}+\cdots+\frac{256}{19683}$

8 terms $= 2^8$

$\dfrac{1/2}{3/4} = 2/3$, $\dfrac{1/3}{1/2} = 2/3$, $\dfrac{2/9}{1/3} = 2/3$

Geometric

common ratio = $2/3$

terms = 11

closed form = $\dfrac{\frac{3}{4}-\frac{512}{59049}}{1-2/3} \approx 2.22$

5. $\frac{1}{4}+\frac{1}{2}+\frac{3}{4}+1+\cdots+\frac{17}{4}$

Not Geometric

6. $e+1+\frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}$

Geometric

common ratio = $1/e$

terms = 5

closed form = $\dfrac{e-1/e^4}{1-1/e} = 4.27$

7. $7-14+28-56+\cdots+114688$

$114688 = 7 \cdot 2^n$

$16384 = 2^n$

$\ln 16384 = n \ln 2$

$n = \dfrac{\ln 16384}{\ln 2} = 14$

Geometric

common ratio = -2

terms = 15

closed form = $\dfrac{7+229376}{1+2} = 76461$

8. $1.2+0.96+0.768+0.6144+0.49152$

$\left\{ \dfrac{0.96}{1.2} = \dfrac{0.768}{0.96} = \dfrac{0.6144}{0.768} = \dfrac{0.49152}{0.6144} = .8 \right.$

Geometric

common ratio = 0.8

terms = 5

closed form = $\dfrac{1.2-0.393216}{1-0.8} = 4.03392$