Math Xa - 1st Exam (1994)

1.

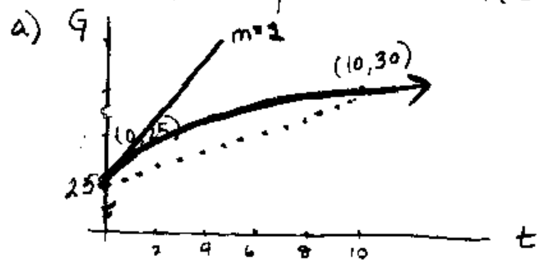
f	f'
a	v
b	viii
c	i

2. i) graph C
ii) graph A
iii) graph D

3. a) x_9 b) x_3 c) x_8 d) x_2

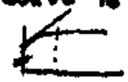
4. From info in problem: $G(0) = 25$ $G(10) = 30$ $G'(0) = 1$

a)



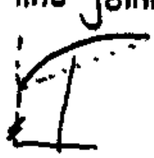
b) Average rate of change over $[0,10]$: $\frac{\Delta G}{\Delta t} = \frac{30-25}{10-0} = \frac{5}{10} = .5$ lbs/day

c) $m_{tan} = 1$. The curve is concave down, so the tangent line lies above the curve.



The point on the tangent line at $t=4$ has a ht. of $25+4\cdot 1 = 29$ so 29 is an upper bound for his weight.

The secant line joining $(0,25)$ and $(10,30)$ lies below the curve.



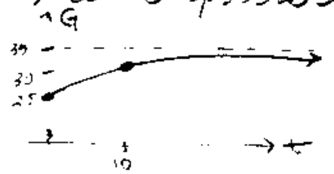
$m_{sec} = .5$ so the pt. on the secant line at $t=4$ has a ht. of $25 + .5(4) = 25 + 2 = 27$

Garfield's weight at $t=4$ is between 27 lbs and 29 lbs

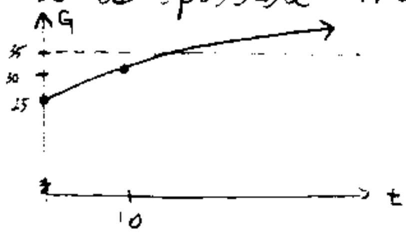
lower bound upper bound

Math X_2 (1994) Solutions (cont)

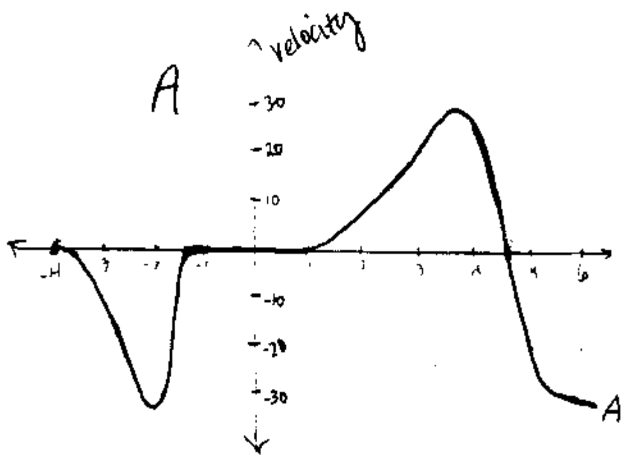
4. a) Yes, it is possible that his weight will remain below 35



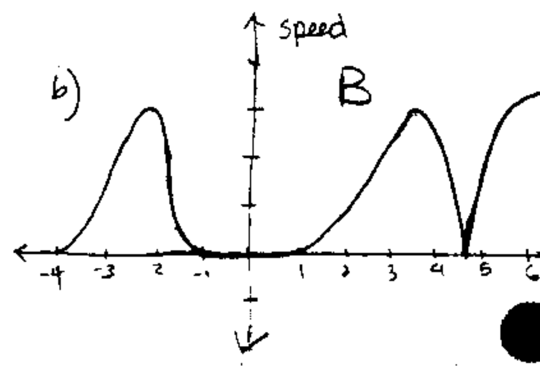
Yes, it is possible that his weight will surpass 35 lbs.



5. a) A



b) B



c) i) a   ii) c

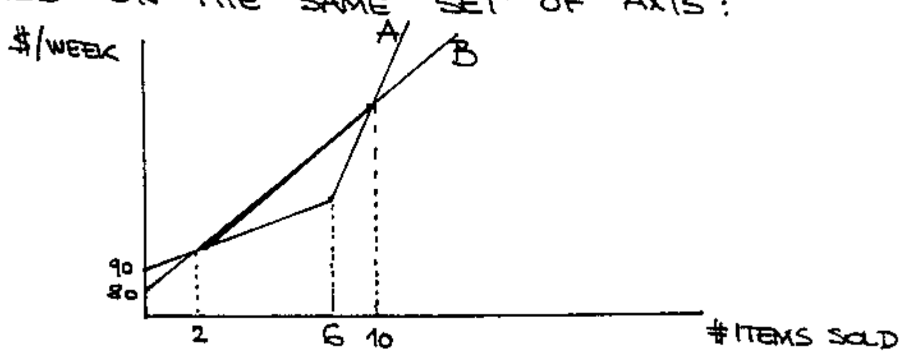
SOLUTIONS OF MATH Xa FIRST EXAM (NOVEMBER 1994)

PROBLEMS 6 – 11

6

a) WEEKLY SALARY IS BASED ON THE NUMBER OF ITEMS SOLD. THERE IS FIXED SALARY EVEN IF NO ITEMS ARE SOLD ($90/WEEK – $ INTERCEP IF LESS THAN 6 ITEMS ARE SOLD SALARY IS INCREASING AT THE RATE $\frac{120-90}{6} = 5$ DOLLARS PER ANY ADDITIONAL ITEM SOLD. IF MORE TI 6 ITEMS ARE SOLD SALARY IS INCREASING AT THE RATE $\frac{150-120}{2}$ DOLLARS PER ANY ADDITIONAL ITEM SOLD.

b) IF COMPANY B IS TO WORKER'S ADVANTAGE, THAT MEANS THAT WORKER IS MAKING MORE MONEY IN COMPANY B THAN IN COMPANY A. TO DETERMINE WHEN IS THIS HAPENING IT IS USEFUL TO GRAPH BO SALARY SCHEMES ON THE SAME SET OF AXIS:



BOTH GRAPHES ARE LINEAR (STRAIGHT LINES) AND POINTS OF INTERSECTI WERE DETERMINED BY SOLVING LINEAR EQUATIONS.

GRAPH A CONSISTS OF 2 LINES – FOR BOTH WE KNOW EITHER ONE POINT AND Y-INTERCEPT OR TWO POINTS. THE SAME IS FOR GRAPH :

$$A: \begin{cases} Y = 5X + 90 & X \le 6 \\ Y = 15X + 30 & X > 6 \end{cases} \qquad B: \ Y = 10X + 80$$

FROM THE GRAPH WE READ THAT WORKER HAS TO SELL MORE THAN 2 AND LESS THAN 10 ITEM PER WEEK IN ORDER TO MAKE MORE MONEY IN COMPANY B.

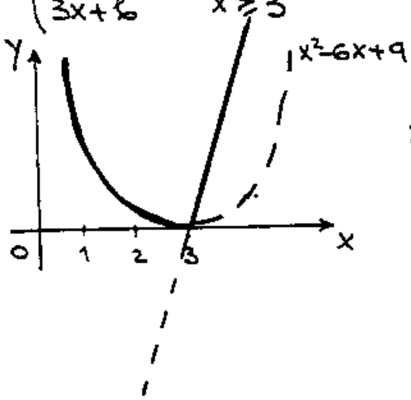
(7) $f(x)=\frac{x}{x-5}$

$$f'(3)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{\frac{3+h}{3+h-5}-\frac{3}{3-5}}{h}=\lim_{h\to 0}\frac{\frac{3+h}{h-2}+\frac{3}{2}}{h}=$$

$$\lim_{h\to 0}\frac{\frac{6+2h+3h-6}{2(h-2)}}{h}=\lim_{h\to 0}\frac{5\cancel{h}}{\cancel{h}\cdot 2\cdot(h-2)}=-\frac{5}{4}$$

(8) $f(x)=\begin{cases}(x-3)^2 & x<3\\ 3x+b & x\geq 3\end{cases}$ $\quad \lim_{x\to 3} f(x)$ EXISTS

a)



$\lim_{x\to 3} f(x)$ WILL EXIST ONLY IF $\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)$

THIS MEANS THAT LINE $y=3x+b$ HAS TO GO THR

POINT $(3,0)$:

$0=3\cdot 3+b$ $\boxed{b=-9}$

b) $f'(6)=3$ BECAUSE $f(6)$ IS ON THE PART OF FINAL GRAPH WHERE $f(x)=3$

c) $f'(0)=-6$ BECAUSE $f(0)$ is ON THE PARABOLA $y=x^2-6x+9$ $(y'=2x-6)$

d) $f'(3)$ DOESN'T EXIST : $\lim_{x\to 3^-} f'(x)=0 \neq \lim_{x\to 3^+} f'(x)=3$

(9) a) $H(t)=H_0\cdot(1.2)^{t/4}$ t IN YEARS

b) i) $\perp$ (GROWTH IS NOT LINEAR)

ii) $\perp$ (PLUG $t=1$)

iii) T (-"-)

c) REWRITE FORMULA FOR $H(t)$

$H(t)=H_0\cdot(1.2)^{t/48}$ t IN MO

(NOTE : AFTER 4 YEARS ($4\cdot 12=48$ MO GROWTH IS 20%)

EVERY MONTH SIZE INCREASES B

$\left[(1.2)^{1/48}=1.003\right]$ 0.3%

(10) $f(x)=x^x$

$(2,2+h)$ $\quad \frac{f(2+h)-f(2)}{h}=\frac{(2+h)^{2+h}-2^2}{h}$ $\underline{\underline{\text{plug } h=0.001}}$ 6.77 – upper BOUND

$f'(2)\approx 6.7$

$(2-h,2)$ $\quad \frac{f(2)-f(2-h)}{h}=\frac{2^2-(2-h)^{2-h}}{h}$ $\underline{\underline{\text{plug } h=0.001}}$ 6.76 – LOWER BOUND

(11) h HOURS A DAY → h/x hours ON THE ROAD
→ $h-h/x$ hours in consultation

Profit = ($/hr consult)(hrs. in consult) – ($ on gas + $ on car)

PROFIT = $A\cdot\left(h-\frac{h}{x}\right)-\left(G+\frac{C}{100}\right)$