1. (a) i.

$$\begin{aligned}(w,z)\in(X\cap Y)\times Z &\iff w\in X\cap Y\\ &\iff w\in X \text{ and } w\in Y\\ &\iff (w,z)\in X\times Z \text{ and } (w,z)\in Y\times Z\\ &\iff (w,z)\in (X\times Z)\cap(Y\times Z)\end{aligned}$$

Hence, $(X\cap Y)\times Z=(X\times Z)\cap(Y\times Z)$.

ii.

$$\begin{aligned}x\in Z\setminus(Y\cup Z) &\iff x\in Z, x\notin (Y\cup Z)\\ &\iff x\in X, y\notin Y \text{ and } x\notin Z\\ &\iff x\in X, x\notin Y \text{ and } x\in X, x\notin Z\\ &\iff x\in X\setminus Y \text{ and } x\in X\setminus Z\\ &\iff x\in (X\setminus Y)\cap(X\setminus Z)\end{aligned}$$

Hence, $X\setminus(Y\cup Z)=(X\setminus Y)\cap(X\setminus Z)$.

iii. $x\in X\setminus(Y\cap Z) \iff x\in X, x\notin(Y\cap Z)$. Since $x\notin(Y\cap Z)$, we know that $x\notin Y$, which holds iff $x\in X\setminus Y$, or $x\notin Z$, which is true iff $x\in X\setminus Z$. So x is either in $X\setminus Y$ or x is in $X\setminus Z$. Hence, $x\in(X\setminus Y)\cup(X\setminus Z)$, which implies that $X\setminus(Y\cap Z)=(X\setminus Y)\cup(X\setminus Z)$.

(b) i.

$$\begin{aligned}\alpha\in f^{-1}(Y\cap Z) &\iff f(\alpha)\in Y\cap Z\\ &\iff f(\alpha)\in Y \text{ and } f(\alpha)\in Z\\ &\iff \alpha\in f^{-1}(Y) \text{ and } \alpha\in f^{-1}(z)\\ &\iff \alpha\in f^{-1}(Y)\cap f^{-1}(Z)\end{aligned}$$

Hence, $f^{-1}(Y\cap Z)=f^{-1}(Y)\cap f^{-1}(Z)$.

ii.

$$\begin{aligned}b\in f^{-1}(Y\setminus Z) &\iff f(b)\in Y\setminus Z\\ &\iff f(b)\in Y, f(b)\notin Z\\ &\iff b\in f^{-1}(Y), b\notin f^{-1}(Z)\\ &\iff b\in f^{-1}(Y)\setminus f^{-1}(Z)\end{aligned}$$

And so $f^{-1}(Y\setminus Z)=f^{-1}(Y)\setminus f^{-1}(Z)$.

(c) i. The statement is true. Take x in $Y\cup Z$ and observe that $f(x)=f(Y\cup Z)$ means that, if $x\in Y$, then $f(x)\in f(Y)$, or if $x\in Z$, then $f(x)\in f(Z)$. Thus, $f(x)\in f(Y\cup Z) \iff f(x)\in f(Y)\cup f(Z)$, which means that $f(Y\cup Z)=f(Y)\cup f(Z)$.

ii. The statement is false. Counterexample via Hilary Havens: Take $f(x) = x^2$ and let $Y = \{-1\}, Z = \{1\}$. Then $Y \cap Z = \emptyset$ and $f(Y \cap Z) = \emptyset$, but $f(Y) \cap f(Z) = 1$, which is a contradiction. Hence $f(Y \cap Z) \neq f(Y) \cap f(Z)$.

(d) i. We'll first show that given a function $f \in (A^B)^C$, we can find a corresponding $h \in A^{B\times C}$. So take $f \in (A^B)^C, c_i \in C, b_j \in B, a_k \in A$. By definition, $f : C \longrightarrow A^B$, and so $f : c_i \longrightarrow g_i, g_i \in G$ the set of all maps g that send B to A. Thus, $g_i(b_j) = a_k$, and given (c_i, b_j), we can get an $a_k \in A$, which means that the desired function h is one such that $h(c_i, b_j) = a_k$, for $h \in A^{B\times C}$.
Now, going in the other direction, given an $h \in A^{B\times C}$, we again take $c_i \in C, b_j \in B, a_k \in A$ such that $h(c_i, b_j) = a_k$. Given the set of all maps g from B to A, we know there exists one such g_i with $g_i(b_j) = a_k$. We then create a map f such that $f(c_i) = g_i$, and we have $f \in (A^B)^C$.
Therefore we have a bijection between $(A^B)^C$ and $A^{B\times C}$.