SOLUTION SET 3A

ALEX LEVIN

Math 23a
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2. Suppose we have $\{w_1, w_2, \ldots, w_{n-1}\}$ that spans V. Then, by a theorem we proved on October 13 and 15, any set of more than $n-1$ vectors cannot be linearly independent. In particular, $\{v_1, \ldots, v_n\}$ is not linearly independent, and we have a contradiction.

4. For $i = 1, 2, \ldots, n$, define the function $\delta_i : S \to \mathbb{R}$ (where $S = \{1, \ldots, n\}$) by:

$$\delta_i(j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

We claim that $\{\delta_1, \delta_2, \ldots, \delta_n\}$ forms a basis for $V = \{f : S \to \mathbb{R}\}$.

To prove liner independence, suppose $a_1\delta_1 + a_2\delta_2 + \ldots + a_n\delta_n = \vec{0}$ (the zero function). However, $(a_1\delta_1 + a_2\delta_2 + \ldots + a_n\delta_n)(1) = a_1\delta_1(1) + a_2\delta_2(1) + \ldots + a_n\delta_n(1) = a_1$, and in general $(a_1\delta_1 + a_2\delta_2 + \ldots + a_n\delta_n)(i) = a_i$. So, for this linear combination to be equal to the zero function, all the a_i's have to be equal to 0.

Now we turn to showing that the δ_i's span V. Given $f \in V$ such that $f(1) = b_1, f(2) = b_2, \ldots, f(n) = b_n$, $f = b_1\delta_1 + b_2\delta_2 + \ldots + b_n\delta_n$. Indeed, $(b_1\delta_1 + b_2\delta_2 + \ldots + b_n\delta_n)(i) = b_1\delta_1(i) + b_2\delta_2(i) + \ldots + b_n\delta_n(i) = b_i = f(i) \ \forall i \in S$.

The δ_i's are called Kronecker delta functions.