MATH 23A 1ST MIDTERM EXAM, OCT 19

Good luck!

There are 50 points total; further breakdown is by question.

1. (8 points) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Find all 3×3 matrices such that $AB = BA$.
2. (10 points) Let $V = \mathbb{R}^3$, $\{e_1, e_2, e_3\}$ be the standard basis of V, and let $v_1 = e_1 + e_2, v_2 = e_2 + e_3, v_3 = e_3$. Let $\{e^1, e^2, e^3\}, \{v^1, v^2, v^3\} \subset V^\vee = V^*$ be the dual bases. Write v^2 as a linear combination of e^1, e^2, e^3.
3. (12 points) Let $\mathbb{C}$ be the set of complex numbers.
   (a) (3 points) Show that the operations of addition of complex numbers and multiplication by a real number a define a structure of a vector space [over $\mathbb{R}$] on $\mathbb{C}$.
   (b) (3 points) Show that $\{1, i\}$ is a basis of $\mathbb{C}$ as a real vector space.
   (c) (3 points) Let $z = a + bi, w = c + di$ (with $a, b, c, d \in \mathbb{R}$) be complex numbers. We define the product $z \cdot w$ of z and w by $z \cdot w = (ac - bd) + (ad + bc)i$. Show that for any $z \in \mathbb{C}$ the map $A(z)\colon \mathbb{C} \to \mathbb{C}$ given by $w \mapsto z \cdot w$ is linear.
   (d) (3 points) Write the matrix of $A(z)$ in respect to the basis $\{1, i\}$ of $\mathbb{C}$.
4. (10 points) Let A, B be two 2×2 matricies such that $AB = 0$. Does this equality imply that $BA = 0$?
5. (10 points)
   (a) (5 points) Let V be a vector space of dimension d. Prove that one can not find a strictly increasing sequence $L_1 \subsetneq L_2 \subsetneq \cdots \subsetneq L_r$ for $r > d + 1$, where the L_i are subspaces.
   (b) (5 points) Let A be a 2×2 matrix such that $A^5 = 0$. Prove that $A^2 = 0$