MATH 23A PROBLEM SET 11

Let V be a finite-dimensional vector space over $\mathbb{R}$.We denote by $C^\infty(V)$ the space of functions f on V such that f has continuous derivatives of all orders. In particular we denote by $C^\infty(\mathbb{R})$ the space of functions f on $\mathbb{R}$ such that f has continuous derivatives of all orders. If $f \in C^\infty(\mathbb{R}), a \in \mathbb{R}$ we define the Taylor series $S_f(a,x)$ for f at a by $S_f(a,x) = \sum_{n\geq 0} f^{(n)}(a)/n!(x-a)^n$.

1.Prove that for any $a, x \in \mathbb{R}$ the Taylor series series $S_f(a,x)$ is convergent to $f(x)$ for $f(x) = \exp(x)$.

2.Let f be a function on $\mathbb{R}$ such that $f(x) = 0$ if $x \leq 0$ and $f(x) = \exp(-1/x)$ for $x > 0$.

a)Show that $f \in C^\infty(\mathbb{R})$.

b)Find the Taylor series $S_f(0,x)$.

c)Explain why does not $S_f(0,x)$ converge to $f(x)$.

3. Let V be a finite-dimensional vector space over $\mathbb{R}$, $f \in C^\infty(V), v \in V$.

a)Define the linear Taylor polynomial $P_1(v,x), x \in V$ for f.

b))Define the quadratic Taylor polynomial $P_2(v,x), x \in V$ for f.

c))Define the Taylor polynomial $P_n(v,x), x \in V$ for f.