Mathematics 22a Homework Problems 9

The following problems are due on **Wednesday, December 2**. Please read B & S, pages 175 - 178. Skim 178- 183. Pages 191 - 195 are extremely sophisticated mathematics, so read only if you're feeling ambitious. The section on Kepler motion, pages 195-197, on the other hand, and readable and useful for physicists (though you will not be tested on it). Finally, you should try to read sections 5.5 and 5.6.

1. Problems 3 and 4 from HW 8.

2. Let f and g be functions given below. In each case, calculate the derivative $d(g \circ f)_p$.

a) $f : \mathbf{R} \to \mathbf{R^2}$ given by the formula $f(x) = (x, \sin(x))$ and $g : \mathbf{R}^2 \to \mathbf{R}$ given by the formula $g(x, y) = x^3y^5$ at the point $p = \pi$.

b) $f : \mathbf{R}^2 \to \mathbf{R}$ given by the formula $f(x, y) = x^2 - y^2$ and $g : \mathbf{R} \to \mathbf{R}^2$ given by the formula $g(x) = (\sin(x), \cos(x))$ at the point $p = (0, 0)$.

c) $f : \mathbf{R} \to \mathbf{R}^2$ given by the formula $f(x) = (e^x, e^{-x})$ and $g : \mathbf{R}^2 \to \mathbf{R}^2$ given by the formula $g(x, y) = ((x + y)/2, (x - y)/2)$ at the point $p = 1$.

d) $f : \mathbf{R^2} \to \mathbf{R}^2$ given by the formula $f(x, y) = (x^3 - xy^2, x^2y - y^3)$ and $g : \mathbf{R}^2 \to \mathbf{R}^2$ given by the formula $g(x, y) = (y^5, -x^5)$ at the point $p = (1, 1)$.

4. Let $f : V \to V$ be a function that is differentiable at the point $p \in V$. Suppose that f has an inverse function $g : V \to V$ (so that $g \circ f(x) = f \circ g(x) = x, \forall x \in V$) that is differentiable at the point $f(p)$. Using the chain rule, express the derivative $dg_{f(p)}$ in terms of the derivative df_p.

6. B & S p. 215, # 5.5.

7. B & S p. 215, # 5.7.