Mathematics 22a Homework Problems 8

The following problems are due on **Monday, November 23**.

1. A line l through the origin in Euclidean space $\mathbf{R}^2$ is uniquely determined by a single vector $\begin{pmatrix} a \\ b \end{pmatrix}$ in the following way: the line l is the set of all points $\begin{pmatrix} x \\ y \end{pmatrix}$ orthogonal (or perpendicular) to $\begin{pmatrix} a \\ b \end{pmatrix}$. This description then gives an equation for the line, namely,

$$< \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} a \\ b \end{pmatrix} >= 0,$$

or in other words,

$$ax + by = 0.$$

Similarly, a *plane* through the origin in $\mathbf{R}^3$ is uniquely determined by a vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$. The plane is the set of all points $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ orthogonal to $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

a) Write the equation of the plane in $\mathbf{R}^3$ that is orthogonal to the vector $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$.

b) Let P be the plane in $\mathbf{R}^3$ defined by the equation $2x - y + 4z = 0$. Find all vectors u in $\mathbf{R}^3$ such that $\|u\| = 1$ and u is orthogonal to P.

c) Let P_1 and P_2 be planes through the origin in $\mathbf{R}^3$. Define the *angle* between P_1 and P_2 by first fixing a vector v_1 lying in P_1 and calculating the minimum angle it forms with a vector v_2 in P_2 to obtain a number (depending on v_1) and then take the maximum such angle over all v_1 lying in P_1. Another way to measure the angle between two planes is to take every possible pair of vectors u_1 and u_2 where u_1 is *orthogonal* to P_1 and u_2 is *orthogonal* to P_2 and then measure the angle between u_1 and u_2. Verify (by drawing a picture, or by a proof) that these two ways of measuring the angle give the same answer.

e) Let P_1 be a plane in $\mathbf{R}^3$ given by the equation $2x - y + 4z = 0$ and let P_2 in $\mathbf{R}^3$ be given by $x + 5y + z = 0$. Calculate the angle between P_1 and P_2.

2. B & S, p. 174, #4.22.

3. Find the derivative df_p of the following functions. Sketch the graph of f where possible (i.e. in a, b, and d).

a) $f : \mathbf{R} \to \mathbf{R}$ given by the formula $f(x) = \sin(\cos(x))$ at the point $p = \pi/2$.
b) $f : \mathbf{R} \to \mathbf{R}^2$ given by the formula $f(x) = (\sin(2x), \cos(x))$ at the point $p = 0$.
c) $f : \mathbf{R}^2 \to \mathbf{R}^2$ given by the formula $f(x, y) = (x^2 - y^2, 2xy)$ at the point $p = (0, 0)$.
d) $f : \mathbf{R}^2 \to \mathbf{R}$ given by the formula $f(x, y) = \log(x^2 + 2y^2 + 1)$ at the point $p = (0, 0)$.

4. 5.4 on p. 215.