Mathematics 22a Homework Problems 7

The following problems are due on **Monday, November 9**.

1. B & S, p. 168, #4.4.

2. B & S, p. 169, #4.5.

3. B & S, p. 169, #4.6.

4. B & S, p. 170 #4.8.

5. Show that the vectors $\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ form a basis for $\mathbf{R}^3$, and apply the Gram-Schmidt process to them.

6. Let $V := \{f : [0,1] \to \mathbf{R} \,|\, f \text{ is continuous}\}$ be the set of continuous functions from $[0,1]$ to $\mathbf{R}$.

a) Prove that V is a vector space under the ordinary addition and scalar multiplication (i.e. $(f+g)(x) = f(x) + g(x)$, $(cf)(x) = cf(x)$).

b) Suppose we define the quantity $< f, g >$ by:

$$< f, g > = \int_0^1 f(t) g(1-t)\, dt.$$

Then is $< \cdot, \cdot >$ bilinear? Symmetric? Positive definite? Why or why not?

c) If $f(x) = x^2$ and $g(x) = x$, find $< f, g >$.

7. Given a vector space V, let V^* be the set of all linear transformations from V to $\mathbb{R}$.

(a) Show that V^* is a vector space.

(b) Let $\{v_1, \dots, v_n\}$ be a basis for V. Define $v_1* : V \to \mathbb{R}$ by the rule: $v_1^*(\sum_k a_k v_k) = a_1$ (for a_i real numbers). Show that v_1^* is in V^*.

(c) Show that $\{v_1^*, dots, v_n^*\}$ is a basis for V^*. This basis is called the dual basis.

(d) Let $f : V \to W$ be a linear transformation. Let $f^* : W^* \to V^*$ be defined as in the class. If $V = W = \mathbb{R}^2$ and A is the matrix of f (with respect to the usual basis), show that A^T is the basis of f^* (with respect to the dual basis).