Mathematics 22a Homework Problems 6

The following problems are due on **Monday, November 4**.

1. B & S, p. 115, #3.10.

2. B & S, p. 117, #3.15.

3. B & S, p. 117, #3.17.

Note: A matrix A has a *square root* B if $B^2 = A$. It is *not* in general true that a square root of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $\begin{pmatrix} \sqrt{a} & \sqrt{b} \\ \sqrt{c} & \sqrt{d} \end{pmatrix}$. As an exercise you might want to figure out exactly when this is true.

4. B & S, p. 118 #3.19.

5. B & S, p. 119 #3.21.

6. Let V be the vector space of two-by-two matrices.

(a) What is the dimension of V?

(b) Find a basis β of V, and prove that it is indeed a basis. Hint: Look for matrices with a lot of zeros.

(c) Fix a two-by-two matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Let $f : V \to V$ be defined by $f(M) = A \cdot M$. Show that f is a linear transformation.

(d) Find the matrix of f with respect to the basis β. Hint: A is *not* the right answer.