Mathematics 22a Homework Problems 3

The following problems are due on **Monday, October 12**.

1-3. B & S, p. 74 #2.5, 2.7, 2.14.

4. Write down the matrix which represents the following linear transformations:
a) $f : \mathbb{R}^2 \to \mathbb{R}^2$; $f((4,-3)) = (8,-6)$, $f((0,2)) = (0,1)$.
b) $f : \mathbb{R}^2 \to \mathbb{R}^2$; $f((3,1)) = (1,3)$, $f((2,2)) = (2,2)$.
c) $f : \mathbb{R}^2 \to \mathbb{R}^2$; $f((2,1)) = (3,5)$, $f((-4,-2)) = (-6,-10)$. (Having trouble with this one? Why?)

5. Let V be the vector space of degree 1 polynomials:
$$V = \{p(x) = ax + b| \quad a, b \in \mathbb{R}\},$$
where addition and scalar multiplication are given by:
$$(p+q)(x) = p(x) + q(x); \qquad (r \cdot p)(x) = rp(x).$$
Consider the linear transformation $f : V \to V$ given by:
$$f(ax + b) = -bx + a.$$
For each of the following ordered pairs of vectors $\beta = \{u, v\}$ in V, determine whether β is a basis for V. If it is a basis, then write down the matrix $[f]_\beta$ representing f in the basis β. In each case, compute the trace and determinant of the matrix you get.
a) $\beta = \{3x, 2x\}$
b) $\beta = \{-1, \frac{1}{2}x\}$
c) $\beta = \{x + 1, -3\}$
d) $\beta = \{2x + 1, x + \frac{1}{2}\}$

6. If a linear transformation is represented by the matrix $\begin{pmatrix} 2 & 3 \\ -4 & 1 \end{pmatrix}$ with respect to some basis v_1, v_2 of a certain two-dimensional vector space, what is the matrix representing it with respect to the basis u_1, u_2 of the same vector space, if u_1, u_2 are known to satisfy the equations $v_1 = 2u_1 - u_2$ and $v_2 = u_2 + 2u_1$?

7. Consider $\beta_1 := \{(2,2), (4,-1)\}$, and $\beta_2 := \{(2,0), (0,-1)\}$, two bases of $\mathbb{R}^2$.
a) Find the matrix B that expresses β_2-coordinates in terms of β_1-coordinates.
b) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map such that $[f]_{\beta_2} := \begin{pmatrix} 1 & -4 \\ 3 & 2 \end{pmatrix}$. Find the matrix $[f]_{\beta_1}$.

7. What is the characteristic polynomial of:
- A nilpotent matrix?
- A projection?
- A diagonal matrix?
- A rotation?