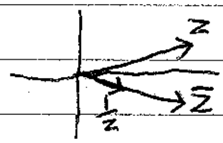
6.4 Solutions

6 If $z=r(\cos\varphi+i\sin\varphi)$, by de Moivre, $z^{-1}=r^{-1}(\cos\varphi-i\sin\varphi)$
$\bar{z}=r(\cos\varphi-i\sin\varphi)=r^2\cdot\frac{1}{z}$



12 If a real quadratic polynomial has complex roots, they must be conjugate because the roots of $a\lambda^2+b\lambda+c$ are $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, and since the imaginary term can only come from $\sqrt{b^2-4ac}$ (a,b,c are real), the other root must be the conjugate of the first.
Since every real polynomial can be factored into real linear and quadratic factors, showing this works for quadratics is sufficient.

24 $\begin{vmatrix}\lambda & -1 & 0\\ 0 & \lambda & -1\\ -5 & 7 & \lambda-3\end{vmatrix}=0$ So $\lambda^3-3\lambda^2+7\lambda-5=0$

$(\lambda-1)(\lambda^2-2\lambda+5)=0$

$(\lambda-1)(\lambda-1-2i)(\lambda-1+2i)=0$

$\lambda=1,\ 1\pm2i$

30a Call the columns of A $\vec{v}_1,\dots,\vec{v}_n$.
Sum of $A\vec{x}$ = sum of $\vec{v}_1x_1$ + sum of $\vec{v}_2x_2$ + ... + sum of $\vec{v}_nx_n$.
But sum of $\vec{v}_i=1$ $\forall i$, since A is a transition matrix.
So sum of $A\vec{x}$ = sum of $\vec{x}=1$.

b By part a) if we raise A to a power, its columns must still sum to 1. Since all non-unity eigenvalues have abs. val. <1, they die when the matrix is exponentiated.
So in the end, we wind up with $\lim_{t\to\infty}A^t$ is a matrix whose columns are all the vector in E_1 whose entries sum to 1.

32a $A = \begin{bmatrix} .6 & .1 & .5 \\ .2 & .7 & .1 \\ .2 & .2 & .4 \end{bmatrix}$

b We just need the eigenvec. of A with eigen val. 1 whose entries sum to 1.
This is $\begin{bmatrix} .4 \\ .35 \\ .25 \end{bmatrix}$, so 40% on AT&T, 35% on MCI, and 25% on Sprint.

38a $C_4^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ $C_4^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ $C_4^4 = I_4$, so $C_4^5 = C_4$, $C_4^6 = C_4^2$, etc.

b $\begin{vmatrix} \lambda & 0 & 0 & -1 \\ -1 & \lambda & 0 & 0 \\ 0 & -1 & \lambda & 0 \\ 0 & 0 & -1 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda^4 - 1 = 0 \Rightarrow \lambda = \pm 1, \pm i.$

$E_1 = \text{span}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $E_{-1} = \text{span}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$ $E_i = \text{span}\begin{pmatrix} 1 \\ -i \\ -1 \\ i \end{pmatrix}$ $E_{-i} = \text{span}\begin{pmatrix} 1 \\ i \\ -1 \\ -i \end{pmatrix}$

c A generic circulant matrix is $aI_4 + bC_4 + cC_4^2 + dC_4^3$
We know from long ago that an eigenvec. of A and B is one of $A+B$ (eigenvals. sum) and AB (eigenvals. multiply).
So eigenval $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = a+b+c+d$ eigenval $\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} = a-b+c-d$

eigenval $\begin{pmatrix} 1 \\ -i \\ -1 \\ i \end{pmatrix} = a+bi-c-di$ eigenval $\begin{pmatrix} 1 \\ i \\ -1 \\ -i \end{pmatrix} = a-bi-c+di$