§2.3 pp. 91–97 # 10, 20, 26, 30, 40, 42 Bryant Mathews

Due Wednesday, Feb. 23

(10) $$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 3 & 6 & 0 & 0 & 1 \end{array}\right) \xrightarrow[-(\mathrm{I})]{-(\mathrm{I})} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 2 & 5 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[-2(\mathrm{II})]{-(\mathrm{II})} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right)$$

$$\xrightarrow[-2(\mathrm{III})]{+(\mathrm{III})} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -3 & 1 \\ 0 & 1 & 0 & -3 & 5 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right), \text{ so}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}^{-1} = \begin{pmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{pmatrix}$$ (invertible)

(20) Write the transformation in matrix form:

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 4 & 8 & 0 & 1 & 0 \\ 2 & 7 & 12 & 0 & 0 & 1 \end{array}\right) \xrightarrow[-2(\mathrm{I})]{-(\mathrm{I})} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & 5 & -1 & 1 & 0 \\ 0 & 1 & 6 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[-(\mathrm{II})]{-3(\mathrm{II})} \left(\begin{array}{ccc|ccc} 1 & 0 & -12 & 4 & -3 & 0 \\ 0 & 1 & 5 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right) \xrightarrow[-5(\mathrm{III})]{+12(\mathrm{III})} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -8 & -15 & 12 \\ 0 & 1 & 0 & 4 & 6 & -5 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right),$$

or
$$\begin{aligned} x_1 &= -8y_1 - 15y_2 + 12y_3 \\ x_2 &= 4y_1 + 6y_2 - 5y_3 \\ x_3 &= -y_1 - y_2 + y_3 \end{aligned}.$$ (invertible)

(26) invertible.

$$\begin{cases} y_1 = x_2 \\ y_2 = x_1^3 + x_2 \end{cases} \iff \begin{cases} x_2 = y_1 \\ x_1^3 = y_2 - y_1 \end{cases} \iff \begin{cases} x_1 = \sqrt[3]{y_2 - y_1} \\ x_2 = y_1 \end{cases}.$$

(30) $\begin{pmatrix} 0 & 1 & b \\ -1 & 0 & c \\ -b & -c & 0 \end{pmatrix} \rightsquigarrow \begin{array}{c} -\mathrm{II} \\ \mathrm{I} \\ \mathrm{III} \end{array} \begin{pmatrix} 1 & 0 & -c \\ 0 & 1 & b \\ -b & -c & 0 \end{pmatrix}$

$\underset{+b(\mathrm{I})}{\rightsquigarrow} \begin{pmatrix} 1 & 0 & -c \\ 0 & 1 & b \\ 0 & -c & -bc \end{pmatrix} \underset{+c(\mathrm{II})}{\rightsquigarrow} \begin{pmatrix} 1 & 0 & -c \\ 0 & 1 & b \\ 0 & 0 & 0 \end{pmatrix}$

only 2 leading ones $\Rightarrow$ never invertible.

(40) Suppose A has two equal columns, $\vec{a}_i$ and $\vec{a}_j$.

Then $A(\vec{e}_i - \vec{e}_j) = \begin{pmatrix} | & & | \\ \vec{a}_i & & \vec{a}_j \\ | & & | \end{pmatrix} \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ -1 \\ \vdots \\ 0 \end{pmatrix} \begin{array}{l} \\ \\ \leftarrow i^{th} \\ \\ \leftarrow j^{th} \\ \\ \\ \end{array} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$

Since $A\vec{x} = \vec{0}$ has a nonzero solution, A is not invertible.

(42) yes, permutation matrices are invertible (by switching rows, we get a rref of I_n).

Yes, the inverse is another permutation matrix. This can be seen by imagining the process of finding the inverse with augmented matrices.